6.59

6.59 The stream function for a certain incompressible, two-dimensional flow field is

$$\psi = 3r^3 \sin 2\theta + 2\theta$$

where ψ is in ft²/s when r is in feet and θ in radians. Determine the shearing stress, $\tau_{r\theta}$, at the point $r = 2$ ft, $\theta = \pi/3$ radians if the fluid is water.

$$\tau_{r\theta} = \mu \left[r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} \right] \qquad \text{(Eq. 6.119d)}$$

With the given stream function,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = 6r^2 \cos 2\theta + \frac{2}{r}$$

and

$$v_\theta = -\frac{\partial \psi}{\partial r} = -9r^2 \sin 2\theta$$

Thus,

$$r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) = -9r \sin 2\theta$$

and

$$\frac{1}{r}\frac{\partial v_r}{\partial \theta} = -12 r \sin 2\theta$$

so that

$$\tau_{r\theta} = \mu\left(-9r \sin 2\theta - 12 r \sin 2\theta\right)$$

$$= -21 \mu r \sin 2\theta$$

(Note that the number 21 would have units of $\frac{1}{ft \cdot s}$.)

For $\mu = 2.34 \times 10^{-5} \frac{lb \cdot s}{ft^2}$, $r = 2\,ft$, and $\theta = \frac{\pi}{3}$ rad,

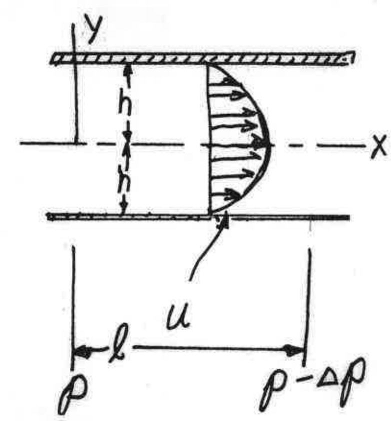
$$\tau_{r\theta} = -\left(21 \frac{1}{ft\cdot s}\right)\left(2.34 \times 10^{-5} \frac{lb\cdot s}{ft^2}\right)\left(2\,ft\right) \sin\frac{2\pi}{3}$$

$$= \underline{\underline{-85.1 \times 10^{-5} \frac{lb}{ft^2}}}$$

59 br

6.65

6.65 Oil ($\mu = 0.4\ N{\cdot}s/m^2$) flows between two fixed horizontal infinite parallel plates with a spacing of 5 mm. The flow is laminar and steady with a pressure gradient of -900 (N/m^2) per unit meter. Determine the volume flowrate per unit width and the shear stress on the upper plate.



Let q = volume flowrate per unit width out of the paper

Thus, with $u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2-h^2)$ (See Eq. 6.127),

$$q = \int_{-h}^{h} u\,dy = \int_{-h}^{h} \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2-h^2)\,dy = \frac{2h^3\Delta p}{3\mu \ell} \quad \text{where } \frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell}$$

For this flow $2h = 5\,mm$ or $h = 2.5\,mm = 2.5\times10^{-3}\,m$

and $\Delta p/\ell = (+900\,N/m^2)/m = +900\,N/m^3$

Thus,

$$q = \frac{2(2.5\times10^{-3}\,m)^3(900\,\frac{N}{m^3})}{3\,(0.4\,N{\cdot}s/m^2)} = \underline{\underline{2.34\times10^{-5}\,\frac{m^2}{s}}}$$

The shear stress is $\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

where

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2-h^2) = -\frac{\Delta p}{2\mu\ell}(y^2-h^2)$$

and

$v = 0$

Hence,

$$\tau_{xy} = -\frac{\Delta p}{2\mu\ell}(2y)\mu = -\frac{\Delta p}{\ell}y$$

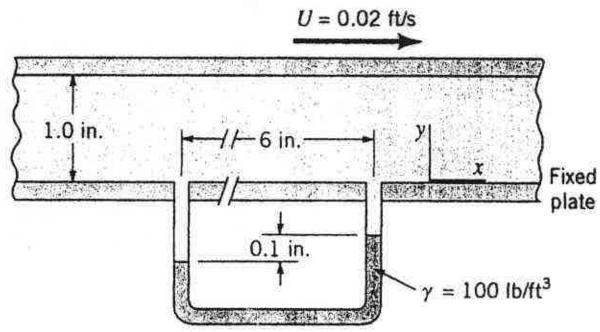
On the upper plate $y=h$ so that

τ_{upper} = magnitude of shear stress on upper plate

$= \frac{\Delta p}{\ell}h = (900\,\frac{N}{m^3})(2.5\times10^{-3}\,m) = \underline{\underline{2.25\,\frac{N}{m^2}}}$ acting in the positive x-direction (the direction of flow).

6.73

6.73 A viscous fluid (specific weight = 80 lb/ft^3; viscosity = 0.03 lb · s/ft^2) is contained between two infinite, horizontal parallel plates as shown in Fig. P6.73. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.1 in. If the upper plate moves with a velocity of 0.02 ft/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.



■ FIGURE P6.73

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by) \qquad \text{(Eq. 6.133)}$$

Maximum velocity will occur at distance y_m where $\frac{du}{dy} = 0$.

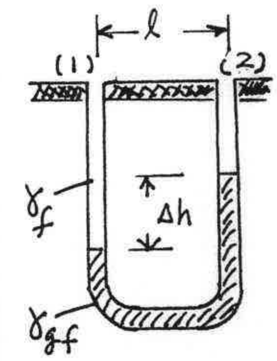
Thus,

$$\frac{du}{dy} = \frac{U}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(2y - b)$$

and for $\frac{du}{dy} = 0$

$$y_m = -\frac{\mu U}{b\left(\frac{\partial p}{\partial x}\right)} + \frac{b}{2} \qquad (1)$$

For manometer (see figure to right),



$$p_1 + \gamma_f \Delta h - \gamma_{gf} \Delta h = p_2$$

or

$$p_1 - p_2 = (\gamma_{gf} - \gamma_f)\Delta h$$

$$= \left(100 \frac{lb}{ft^3} - 80 \frac{lb}{ft^3}\right)\left(\frac{0.1 \text{ in.}}{12 \frac{in.}{ft}}\right) = 0.167 \frac{lb}{ft^2}$$

Also,

$$-\frac{\partial p}{\partial x} = \frac{p_1 - p_2}{\ell} = \frac{0.167 \frac{lb}{ft^2}}{\left(\frac{6 \text{ in.}}{12 \frac{in.}{ft}}\right)} = 0.334 \frac{lb}{ft^3}$$

Thus, from Eq. (1)

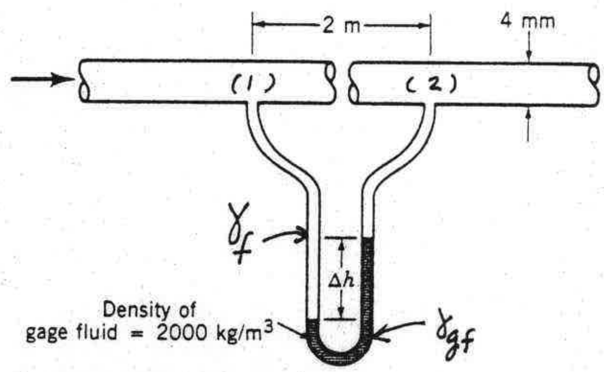
$$y_m = -\frac{\left(0.03 \frac{lb \cdot s}{ft^2}\right)\left(0.02 \frac{ft}{s}\right)}{\left(\frac{1.0 \text{ in.}}{12 \frac{in.}{ft}}\right)\left(-0.334 \frac{lb}{ft^3}\right)} + \frac{\frac{1.0 \text{ in.}}{12 \frac{in.}{ft}}}{2}$$

$$= 0.0632 \text{ ft} \left(\frac{12 \text{ in.}}{ft}\right) = \underline{\underline{0.759 \text{ in.}}}$$

73 br

6.81

6.81 A liquid (viscosity = 0.002 N·s/m^2; density = 1000 kg/m^3) is forced through the circular tube shown in Fig. P6.81. A differential manometer is connected to the tube as shown to measure the pressure drop along the tube. When the differential reading, Δh, is 9 mm, what is the mean velocity in the tube?



■ FIGURE P6.81

Assume laminar flow so that

$$V = \frac{R^2}{8\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.145)}$$

For manometer (see figure),

$$p_1 + \gamma \Delta h - \gamma_{gf}\Delta h = p_2$$

or

$$p_1 - p_2 = \Delta p = \Delta h\,(\gamma_{gf} - \gamma) = \Delta h\,(g)(\rho_{gf} - \rho_f)$$

$$= (0.009\,m)\left(9.81\,\frac{m}{s^2}\right)\left(2000\,\frac{kg}{m^3} - 1000\,\frac{kg}{m^3}\right)$$

$$= 88.3\,\frac{N}{m^2}$$

Thus,

$$V = \frac{\left(\frac{0.004}{2}\,m\right)^2\left(88.3\,\frac{N}{m^2}\right)}{8\left(0.002\,\frac{N\cdot s}{m^2}\right)(2\,m)} = \underline{\underline{1.10\times10^{-2}\,\frac{m}{s}}}$$

Check Reynolds number to confirm that flow is laminar:

$$R_e = \frac{\rho V(2R)}{\mu} = \frac{\left(10^3\,\frac{kg}{m^3}\right)\left(1.10\times10^{-2}\,\frac{m}{s}\right)(0.004\,m)}{0.002\,\frac{N\cdot s}{m^2}}$$

$$= 22.0 < 2100$$

Since $R_e < 2100$ flow is laminar.

81 br