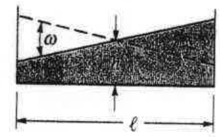
7.5

7.5 Water sloshes back and forth in a tank as shown in Fig. P7.5. The frequency of sloshing, ω, is assumed to be a function of the acceleration of gravity, g, the average depth of the water, h, and the length of the tank, ℓ. Develop a suitable set of dimensionless parameters for this problem using g and ℓ as repeating variables.



■ FIGURE P7.5

$$\omega = f(g, h, \ell)$$

$\omega \doteq T^{-1} \quad g \doteq LT^{-2} \quad h \doteq L \quad \ell \doteq L$

From the pi theorem, $4-2=2$ dimensionless parameters required. Use g and ℓ as repeating variables, Thus,

$$\Pi_1 = \omega g^a \ell^b$$

and

$$(T^{-1})(LT^{-2})^a(L)^b \doteq L^0 T^0$$

so that

$$a + b = 0 \qquad \text{(for } L\text{)}$$

$$-1 - 2a = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1/2$, $b = 1/2$, and therefore

$$\Pi_1 = \omega\sqrt{\frac{\ell}{g}}$$

Check dimensions:

$$\omega\sqrt{\frac{\ell}{g}} \doteq \frac{1}{T}\sqrt{\frac{L}{LT^{-2}}} \doteq L^0 T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = h\, g^a \ell^b$$

$$L(LT^{-2})^a(L)^b \doteq L^0 T^0$$

$$1 + a + b = 0 \qquad \text{(for } L\text{)}$$

$$-2a = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 0$, $b = -1$, and therefore

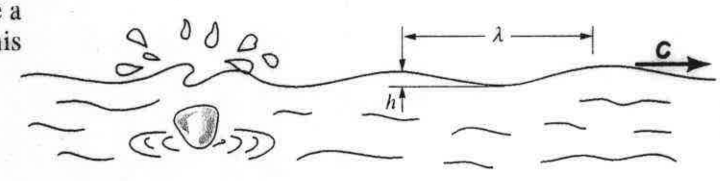
$$\Pi_2 = \frac{h}{\ell}$$

and Π_2 is obviously dimensionless. Thus,

$$\omega\sqrt{\frac{\ell}{g}} = \phi\left(\frac{h}{\ell}\right)$$

7.9

7.9 When a small pebble is dropped into a liquid, small waves travel outward as shown in Fig. P7.9. The speed of these waves, c, is assumed to be a function of the liquid density, ρ, the wavelength, λ, the wave height, h, and the surface tension of the liquid, σ. Use h, ρ, and σ as repeating variables to determine a suitable set of pi terms that could be used to describe this problem.



■ FIGURE P7.9

$$c = f(\rho, \lambda, h, \sigma)$$

$c \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \lambda \doteq L$ $\quad h \doteq L$ $\quad \sigma \doteq FL^{-1}$

From the pi theorem, $5-3 = 2$ pi terms required. Use h, ρ, and σ as repeating variables. Thus,

$$\Pi_1 = c\, h^a \rho^b \sigma^c$$

and $\quad (LT^{-1})(L)^a(FL^{-4}T^2)^b(FL^{-1})^c \doteq F^0L^0T^0$

so that

$$b + c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a - 4b - c = 0 \qquad \text{(for } L\text{)}$$
$$-1 + 2b = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 1/2$, $b = 1/2$, $c = -1/2$, and therefore

$$\Pi_1 = c\, h^{1/2} \rho^{1/2} \sigma^{-1/2} = c\sqrt{\frac{\rho h}{\sigma}}$$

Check dimensions:

$$c\sqrt{\frac{\rho h}{\sigma}} \doteq (LT^{-1})\left[\frac{(FL^{-4}T^2)(L)}{(FL^{-1})}\right]^{1/2} \doteq F^0L^0T^0 \therefore \text{OK}$$

For Π_2: $\quad \Pi_2 = \lambda\, h^a \rho^b \sigma^c$

and $\quad (L)(L)^a(FL^{-4}T^2)^b(FL^{-1})^c \doteq F^0L^0T^0$

so that

$$b + c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a - 4b - c = 0 \qquad \text{(for } L\text{)}$$
$$2b = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1$, $b = 0$, $c = 0$, so that

$$\Pi_2 = \frac{\lambda}{h}$$

which is obviously dimensionless. Thus,

$$\underline{\underline{c\sqrt{\frac{\rho h}{\sigma}} = \phi\left(\frac{\lambda}{h}\right)}}$$

7.12

7.12 At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity, V, in the larger pipe, and the fluid density, ρ, and viscosity, μ. Use D_1, V, and μ as repeating variables to determine a suitable set of dimensionless parameters. Why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

$$\Delta p = f(D_1, D_2, V, \rho, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D_1 \doteq L$ $\quad D_2 \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $6-3=3$ dimensionless parameters required. Use D_1, V, and μ as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D_1^a V^b \mu^c$$

and

$$(FL^{-2})(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$
$$-2 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=1$, $b=-1$, $c=-1$, and therefore

$$\Pi_1 = \frac{\Delta p\, D_1}{V\mu}$$

Check dimensions using MLT system:

$$\frac{\Delta p\, D_1}{V\mu} \doteq \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = D_2 D_1^a V^b \mu^c$$

$$L(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$
$$1 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$
$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{D_2}{D_1}$$

(con't)

7.12

7.12 (Con't)

Π_2 is obviously dimensionless.

For Π_3:

$$\Pi_3 = \rho D_1^{a} V^{b} \mu^{c}$$

$$(FL^{-4}T^{2})(L)^{a}(LT^{-1})^{b}(FL^{-2}T)^{c} \doteq F^{0}L^{0}T^{0}$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-4 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$

$$2 - b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 1$, $b = 1$, $c = -1$ and therefore

$$\Pi_3 = \frac{\rho D_1 V}{\mu}$$

Check dimensions using MLT system:

$$\frac{\rho D_1 V}{\mu} \doteq \frac{(ML^{-3})(L)(LT^{-1})}{ML^{-1}T^{-1}} \doteq M^{0}L^{0}T^{0} \quad \therefore OK$$

Thus,

$$\frac{\Delta p \, D_1}{V\mu} = \phi\left(\frac{D_2}{D_1}, \frac{\rho D_1 V}{\mu}\right)$$

From the continuity equation,

$$V \frac{\pi}{4} D_1^{2} = V_s \frac{\pi}{4} D_2^{2}$$

where V_s is the velocity in the smaller pipe. Since

$$V_s = \left(\frac{D_1}{D_2}\right)^2 V$$

V_s is not independent of D_1, D_2, and V and therefore should <u>not</u> be included as an independent variable.

7.15

7.23*

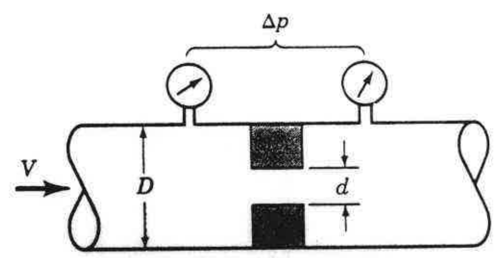
*7.23 The pressure drop across a short hollowed plug placed in a circular tube through which a liquid is flowing (see Fig. P7.23) can be expressed as

$$\Delta p = f(\rho, V, D, d)$$

where ρ is the fluid density, and V is the mean velocity in the tube. Some experimental data obtained with $D = 0.2$ ft, $\rho = 2.0$ slugs/ft^3, and $V = 2$ ft/s are given in the following table:

d (ft)	0.06	0.08	0.10	0.15
Δp (lb/ft^2)	493.8	156.2	64.0	12.6

Plot the results of these tests, using suitable dimensionless parameters, on log–log graph paper. Use a standard curve-fitting technique to determine a general equation for Δp. What are the limits of applicability of the equation?



■ FIGURE P7.23

$\Delta p \doteq FL^{-2} \quad \rho \doteq FL^{-4}T^{2} \quad V \doteq LT^{-1} \quad D \doteq L \quad d \doteq L$

From the pi theorem, $5-3=2$ pi terms required. By inspection for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p}{\rho V^2} \doteq \frac{FL^{-2}}{(FL^{-4}T^{2})(LT^{-1})^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \quad \therefore OK$$

For Π_2 (containing D and d):

$$\Pi_2 = \frac{D}{d}$$

(which is obviously dimensionless). Thus,

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{D}{d}\right)$$

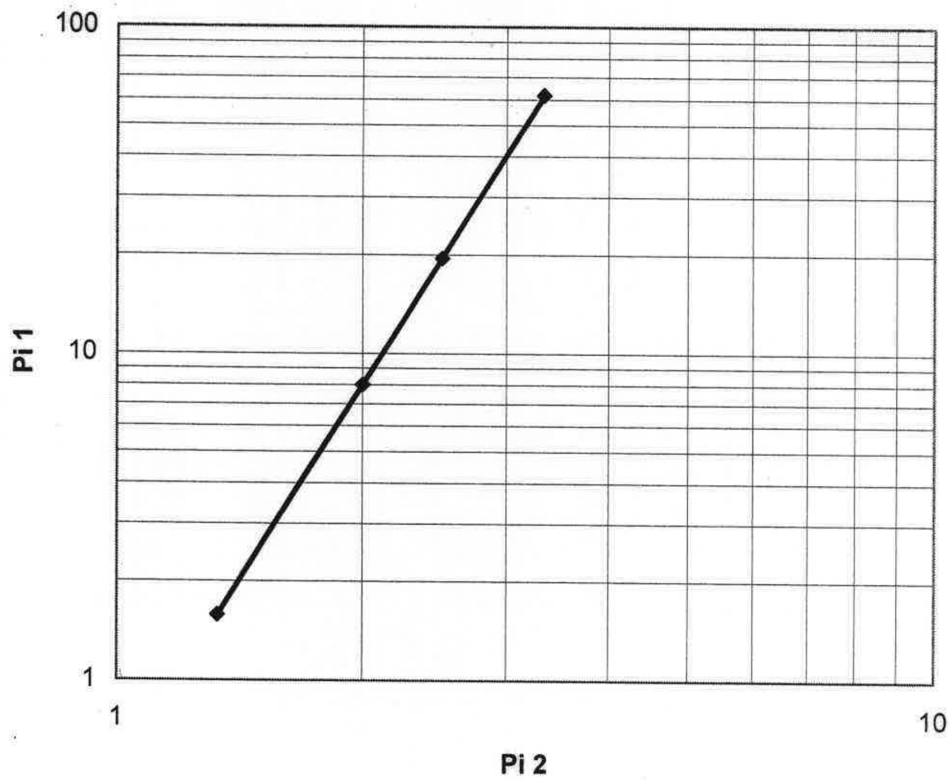
For the data given:

D/d	3.33	2.50	2.00	1.33
$\Delta p/\rho V^2$	61.7	19.5	8.00	1.58

A log-log plot of these data is shown on the following page.

(con't)

7.23*



Since the data plot as a straight line on a log-log plot, the equation for the data is of the form

$$\Pi_1 = a\,\Pi_2^{\,b}$$

where $\Pi_1 = \Delta p/\rho V^2$ and $\Pi_2 = D/d$. A power law fit of the data gives

$$a = 0.505 \quad \text{and} \quad b = 3.99$$

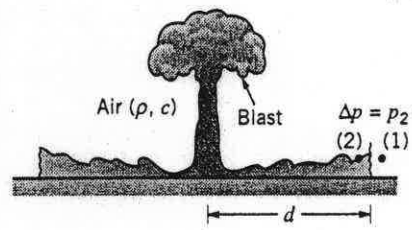
Thus,

$$\frac{\Delta p}{\rho V^2} = 0.505\left(\frac{D}{d}\right)^{3.99}$$

This equation is applicable over the range of data $1.33 \le \frac{D}{d} \le 3.33$.

7.35

7.35 The pressure rise, Δp, across a blast wave, as shown in Fig. P7.35 is assumed to be a function of the amount of energy released in the explosion, E, the air density, ρ, the speed of sound, c, and the distance from the blast, d. **(a)** Put this relationship in dimensionless form. **(b)** Consider two blasts: the prototype blast with energy release E and a model blast with 1/1000th the energy release ($E_m = 0.001\,E$). At what distance from the model blast will the pressure rise be the same as that at a distance of 1 mile from the prototype blast?



■ FIGURE P7.35

(a)
$$\Delta p = f(E, \rho, c, d)$$

$\Delta p \doteq FL^{-2}$ $\quad E \doteq FL$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad c \doteq LT^{-1}$ $\quad d \doteq L$

From the pi theorem, $5-3=2$ pi terms required, and a dimensional analysis yields

$$\underline{\underline{\frac{\Delta p}{\rho c^2} = \phi\left(\frac{E}{\rho c^2 d^3}\right)}}$$

(b) For similarity,

$$\frac{E_m}{\rho_m c_m^2 d_m^3} = \frac{E}{\rho c^2 d^3}$$

and with $\rho_m = \rho$, $c_m = c$, it follows that

$$d_m^3 = \frac{E_m}{E} d^3$$

For $E_m/E = 0.001$ and $d = 1\text{ mi}$

$$d_m^3 = (0.001)(1\text{ mi})^3$$

$$d_m = 0.100\text{ mi}$$

With this similarity requirement satisfied, the prediction equation is

$$\frac{\Delta p_m}{\rho_m c_m^2} = \frac{\Delta p}{\rho c^2}$$

and therefore

$$\Delta p_m = \Delta p$$

at

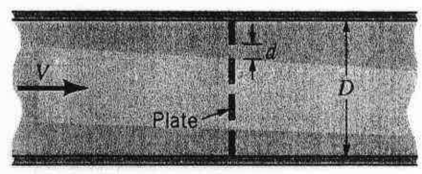
$$\underline{\underline{d_m = 0.100\text{ mi}}}$$

7.35

7.41

7.41 As shown in Fig. P7.41, a thin, flat plate containing a series of holes is to be placed in a pipe to filter out any particles in the liquid flowing through the pipe. There is some concern about the large pressure drop that may develop across the plate, and it is proposed to study this problem with a geometrically similar model. The following data apply.
(a) Assuming that the pressure drop, Δp, depends on the variables listed, use dimensional analysis to develop a suitable set of dimensionless parameters for this problem. **(b)** Determine values for the model indicated in the list with a question mark. What will be the pressure drop scale, $\Delta p_m/\Delta p$?

Prototype	Model
d—hole diameter = 1.0 mm	d = ?
D—pipe diameter = 50 mm	D = 10 mm
μ—viscosity = 0.002 N·s/m^2	μ = 0.002 N·s/m^2
ρ—density = 1000 kg/m^3	ρ = 1000 kg/m^3
V—velocity = 0.1 m/s to 2 m/s	V = ?



■ FIGURE P7.41

(a) $$\Delta p = f(d, D, \mu, \rho, V)$$

$$\Delta p \doteq FL^{-2} \quad d \doteq L \quad D \doteq L \quad \mu \doteq FL^{-2}T \quad \rho \doteq FL^{-4}T^2 \quad V \doteq LT^{-1}$$

From the pi theorem, 6−3 = 3 pi terms required, and a dimensional analysis yields

$$\frac{\Delta p}{\rho V^2} = \phi\left(\frac{d}{D}, \frac{\rho V D}{\mu}\right)$$

(b) For similarity,

$$\frac{d_m}{D_m} = \frac{d}{D}$$

and with the data given

$$d_m = \frac{D_m}{D} d = \left(\frac{10\text{ mm}}{50\text{ mm}}\right)(1.0\text{ mm}) = \underline{0.200\text{ mm}}$$

Also,

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho V D}{\mu}$$

and with $\rho_m = \rho$, $\mu_m = \mu$ it follows that

$$V_m = \frac{D}{D_m} V = \left(\frac{50\text{ mm}}{10\text{ mm}}\right) V = 5V$$

$$= 5\left(0.1\tfrac{m}{s} \text{ to } 2\tfrac{m}{s}\right) = \underline{0.500\tfrac{m}{s} \text{ to } 10.0\tfrac{m}{s}}$$

With the similarity requirements satisfied, the prediction equation is

$$\frac{\Delta p_m}{\rho_m V_m^2} = \frac{\Delta p}{\rho V^2}$$

so that

$$\frac{\Delta p_m}{\Delta p} = \left(\frac{\rho_m}{\rho}\right)\left(\frac{V_m}{V}\right)^2 = (1)(5)^2 = \underline{25.0}$$