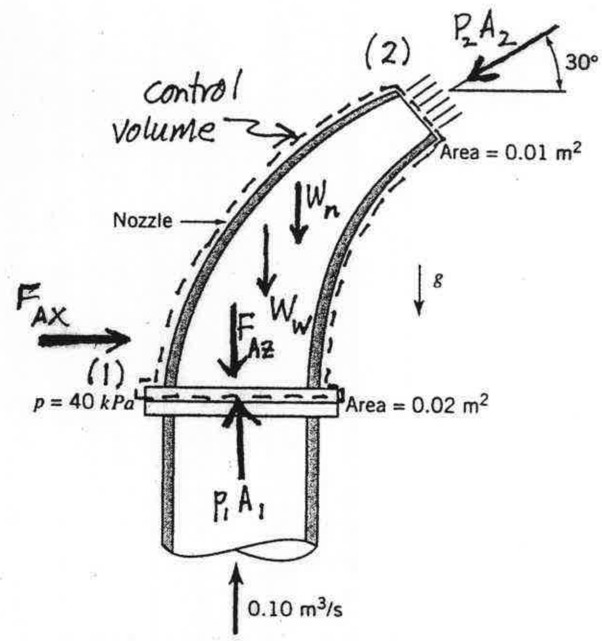
5.26

5.26 A nozzle is attached to a vertical pipe and discharges water into the atmosphere as shown in Fig. P5.26. When the discharge is 0.1 m³/s, the gage pressure at the flange is 40 kPa. Determine the vertical component of the anchoring force required to hold the nozzle in place. The nozzle has a weight of 200 N, and the volume of water in the nozzle is 0.012 m³. Is the anchoring force directed upward or downward?



■ FIGURE P5.26

The analysis leading to the solution of this problem is similar to the one outlined in Example 5.7. Included in the control volume are the nozzle and the water in the nozzle at an instant. Application of the vertical or z-direction component of the linear momentum equation (Eq. 5.17) to the flow through this control volume leads to

$$\dot{m}(V_2 \sin 30° - V_1) = P_1A_1 - F_{AZ} - W_n - W_w - \overbrace{\cancel{P_2}}^{0 \text{ gage}} A_2 \sin 30° \quad (1)$$

Solving Eq. 1 for F_{AZ} yields

$$F_{AZ} = P_1A_1 - W_n - W_w - \dot{m}(V_2 \sin 30° - V_1) \quad (2)$$

For $\dot{m}$ we use $\dot{m} = \rho Q$

For W_w we use $W_w = \forall_w \gamma_w$

(con't)

26 br

5.26

(con't)

Also, we note that $V_1 = \frac{Q_1}{A_1}$

Thus, Eq. 2 becomes

$$F_{Az} = P_1 A_1 - W_n - \forall_w \gamma_w - \rho Q \left(\frac{Q}{A_2} \sin 30^\circ - \frac{Q}{A_1} \right)$$

or

$$F_{Az} = (40\ kPa)\left(1 \frac{N}{m^2 \cdot Pa}\right)\left(1000 \frac{Pa}{kPa}\right)\left(0.02\ m^2\right) - 200\ N$$
$$- \left(0.012\ m^3\right)\left(9.8 \frac{kN}{m^3}\right)\left(1000 \frac{N}{kN}\right)$$
$$- \left(999 \frac{kg}{m^3}\right)\left(0.01 \frac{m^3}{s}\right)\left(1 \frac{N \cdot s^2}{kg \cdot m}\right)\left[\left(\frac{0.01 \frac{m^3}{s}}{0.01\ m^2}\right)\sin 30^\circ - \left(\frac{0.01 \frac{m^3}{s}}{0.02\ m^2}\right)\right]$$

and

$$F_{Az} = 800N - 200N - 117.6N - 0N = \underline{\underline{482\ N}} \text{ downward}$$

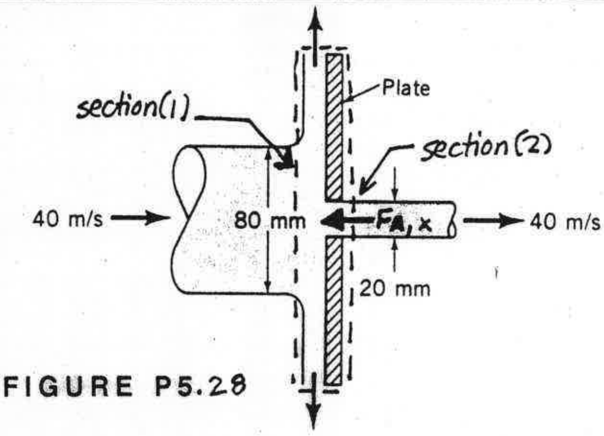
26 br

5.20

21

5.28

5.28 A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown in Fig. P5.28. A hole at the center of the plate results in a discharge jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force required to hold the plate stationary.



■ **FIGURE P5.28**

The control volume contains the plate and flowing air as indicated in the sketch above. Application of the horizontal or x direction component of the linear momentum equation (Eq. 5.17)

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

or

$$F_{A,x} = u_1^2 \rho \frac{\pi D_1^2}{4} - u_2^2 \rho \frac{\pi D_2^2}{4} = u_1^2 \rho \frac{\pi}{4}\left(D_1^2 - D_2^2\right)$$

Thus,

$$F_{A,x} = \left(40 \frac{m}{s}\right)^2 \left(1.23 \frac{kg}{m^3}\right) \frac{\pi}{4}\left[\frac{(80\ mm)^2 - (20\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\right]\left(\frac{1\ N}{kg \cdot \frac{m}{s^2}}\right)$$

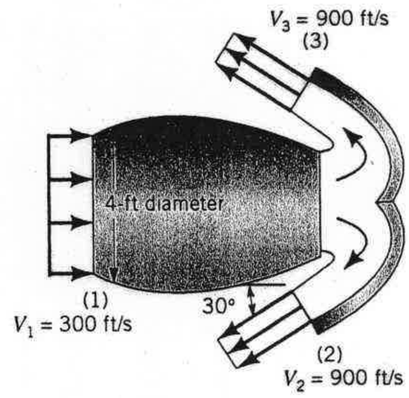
and

$$F_{A,x} = \underline{\underline{9.27\ N}}$$

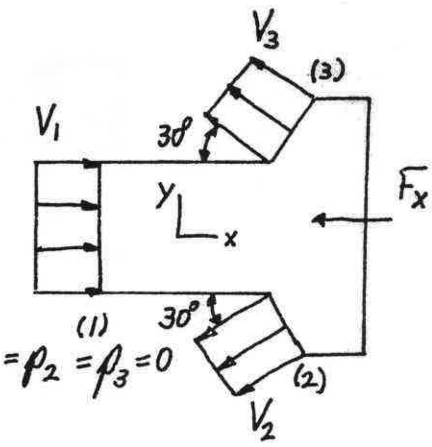
28br

5.29

5.29 (See Fluids in the News article titled **"Where the plume goes,"** Section 5.2.2.) Air flows into the jet engine shown in Fig. P5.29 at a rate of 9 slugs/s and a speed of 300 ft/s. Upon landing, the engine exhaust exits through the reverse thrust mechanism with a speed of 900 ft/s in the direction indicated. Determine the reverse thrust applied by the engine to the airplane. Assume the inlet and exit pressures are atmospheric and that the mass flowrate of fuel is negligible compared to the air flowrate through the engine.



■ **FIGURE P5.29**



$p_1 = p_2 = p_3 = 0$

The momentum equation (x-component), Eq. 5.17,

$\sum_{out} u\rho AV - \sum_{in} u\rho AV = \sum F_x$, for the control volume shown can be written as

$$V_1\rho(-V_1)A_1 + (-V_2\cos 30°)\rho V_2 A_2 + (-V_3\cos 30°)\rho V_3 A_3 = -F_x$$

or

$$F_x = (\rho V_1 A_1)V_1 + (\rho V_2 A_2)V_2\cos 30° + (\rho V_3 A_3)V_3\cos 30° \qquad (1)$$

But from conservation of mass,

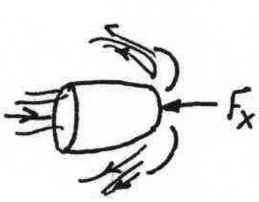
$$\rho V_1 A_1 = \rho V_2 A_2 + \rho V_3 A_3 = \dot{m} = 9\ slugs/s$$

Also, $V_2 = V_3$, so that Eq. (1) becomes

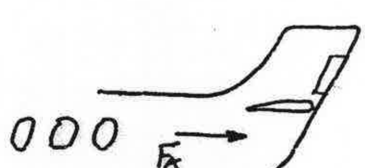
$$F_x = \dot{m}(V_1 + V_2\cos 30°) = 9\,\frac{slugs}{s}\left(300\,\frac{ft}{s} + 900\cos 30°\,\frac{ft}{s}\right)$$

$$= 9710\,\frac{slugs\cdot ft}{s^2} = \underline{\underline{9170\ lb}}$$

Note direction of F_x on engine and engine on airplane.



on engine

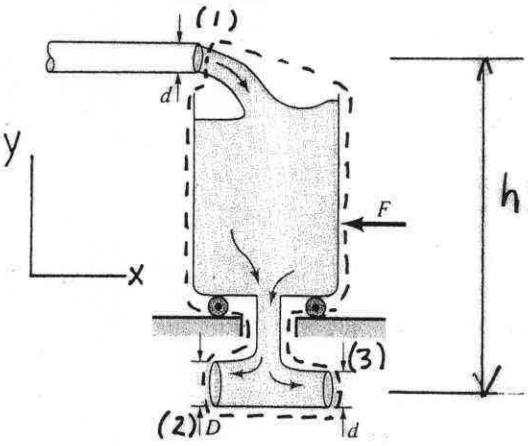


on airplane

29 br

5.31

5.31 Water flows steadily into and out of a tank that sits on frictionless wheels as shown in Fig. P5.31. Determine the diameter D so that the tank remains motionless if $F = 0$.



■ FIGURE P5.31

Applying the horizontal component of the linear momentum equation to the contents of the control volume shown in the sketch we get:

$$\sum u_{out}\,\rho\, V_{out} A_{out} - \sum u_{in}\,\rho\, V_{in} A_{in} = \sum F_x$$

or

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 + V_3 \rho V_3 A_3 = 0$$

and

$$-V_1^2 \rho \frac{\pi d^2}{4} - V_2^2 \rho \frac{\pi D^2}{4} + V_3^2 \rho \frac{\pi d^2}{4} = 0$$

Since $V_2 = V_3 = \sqrt{2gh}$ we obtain

$$V_1^2 d^2 = V_3^2 d^2 - V_3 D^2 \qquad (1)$$

From the conservation of mass equation we get

$$Q_1 = Q_3 + Q_3$$

or

$$V_1 d^2 = V_2 D^2 + V_3 d^2$$

Again, since $V_2 = V_3 = \sqrt{2gh}$ we get

$$V_1 d^2 = V_3 D^2 + V_3 d^2 \qquad (2)$$

Looking at Eqs. (1) and (2) together we conclude

If $V_3 < V_1$ eq. (1) cannot be satisfied
eq. (2) can be satisfied

If $V_3 > V_1$ eq. (1) can be satisfied
eq. (2) cannot be satisfied

If $V_3 = V_1$ eq. (1) can be satisfied with $D = 0$
eq. (2) can be satisfied with $D = 0$

So $V_3 = V_1$ and $\underline{\underline{D = 0}}$

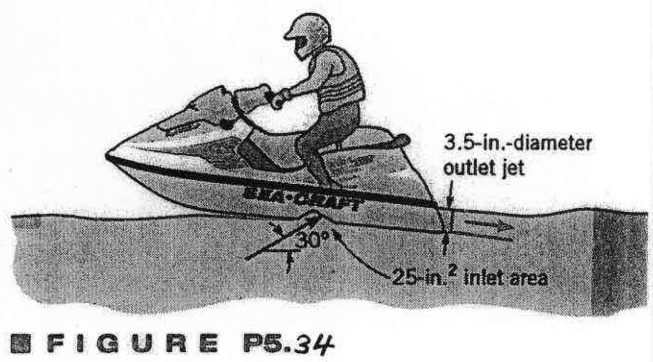
For $V_3 = V_1$ h must be set so that

$$V_3 = \sqrt{2gh} = V_1$$

31 br

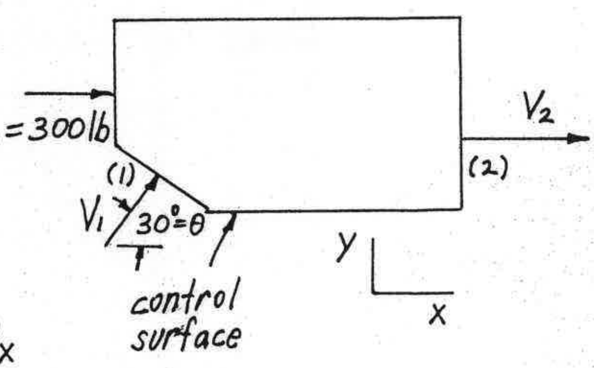
5.34

5.34 The thrust developed to propel the jet ski shown in **Video V9.12** and Fig. P5.34 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300-lb thrust? Assume the inlet and outlet jets of water are free jets.



■ FIGURE P5.34

For the control volume indicated the x-component of the momentum equation



$$\int_{cs} u \rho \vec{V} \cdot \hat{n} \, dA = \sum F_x \quad \text{becomes}$$

(1) $$(V_1 \cos 30°)\rho(-V_1)A_1 + V_2 \rho(+V_2)A_2 = R_x$$

where we have assumed that $p = 0$ on the entire control surface and that the exiting water jet is horizontal.

With $\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ Eq. (1) becomes

$$R_x = \dot{m}(V_2 - V_1 \cos\theta) = \rho V_1 A_1 (V_2 - V_1 \cos 30°) \qquad (1)$$

Also, $A_1 V_1 = A_2 V_2$ so that

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{25 \text{ in.}^2}{\frac{\pi}{4}(3.5 \text{ in.})^2} V_1 = 2.60 \, V_1 \qquad (2)$$

By combining Eqs. (1) and (2):

$$R_x = \rho V_1^2 A_1 (2.60 - \cos 30°)$$

or

$$V_1 = \left[\frac{300 \text{ lb}}{\left(1.94 \frac{\text{slugs}}{\text{ft}^3}\right)\left(\frac{25}{144} \text{ ft}^2\right)(2.60 - \cos 30°)}\right]^{1/2} = 22.7 \frac{\text{ft}}{\text{s}}$$

Thus,

$$Q = A_1 V_1 = \left(\frac{25}{144} \text{ ft}^2\right)\left(22.7 \frac{\text{ft}}{\text{s}}\right) = \underline{\underline{3.94 \frac{\text{ft}^3}{\text{s}}}}$$

34 bi

5.37

5.37 A horizontal circular cross section jet of air having a diameter of 6 in. strikes a conical deflector as shown in Fig. P5.37. A horizontal anchoring force of 5 lb is required to hold the cone in place. Estimate the nozzle flow rate in ft³/s. The magnitude of the velocity of the air remains constant.

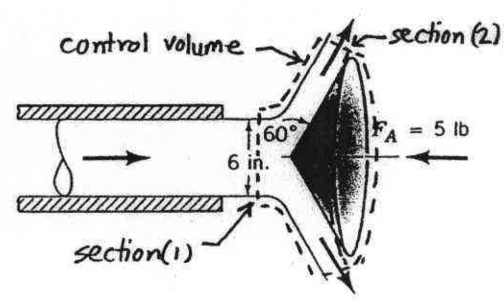


FIGURE P5.37

The control volume shown in the sketch is used. Application of the axial or x-direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

With the conservation of mass principle we can conclude for this incompressible flow that

$$u_1 A_1 = u_2 A_2 = Q$$

Also

$$u_2 = V \cos 60^\circ$$

and

$$u_1 = V = \frac{Q}{A_1}$$

Thus

$$-V\rho Q + V\cos 60^\circ \rho Q = -F_{A,x} = -\frac{Q^2}{A_1}\rho + \frac{Q^2}{A_1}\cos 60^\circ \rho$$

or

$$Q = \left[\frac{F_{A,x}\, A_1}{\rho(1-\cos 60^\circ)}\right]^{\frac{1}{2}} = \left[\frac{F_{A,x}\left(\frac{\pi D_1^2}{4}\right)}{\rho(1-\cos 60^\circ)}\right]^{\frac{1}{2}}$$

Thus

$$Q = \left[\frac{(5\text{ lb})(\pi)(6\text{ in.})^2}{\left(0.00238\frac{\text{slugs}}{\text{ft}^3}\right)(1-\cos 60^\circ)(4)\left(144\frac{\text{in.}^2}{\text{ft}^2}\right)\left(\frac{1\text{ lb}}{\text{slug}\frac{\text{ft}}{\text{s}^2}}\right)}\right]^{\frac{1}{2}}$$

and

$$Q = \underline{\underline{28.7}}\ \frac{\text{ft}^3}{\text{s}}$$

37 bi

5.53

5.53 Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.53. The exit cross section area of each of the two nozzles is 0.04 in.^{2} and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. **(a)** Determine the resisting torque required to hold the sprinkler head stationary. **(b)** Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. **(c)** Determine the angular velocity of the sprinkler if no resisting torque is applied.

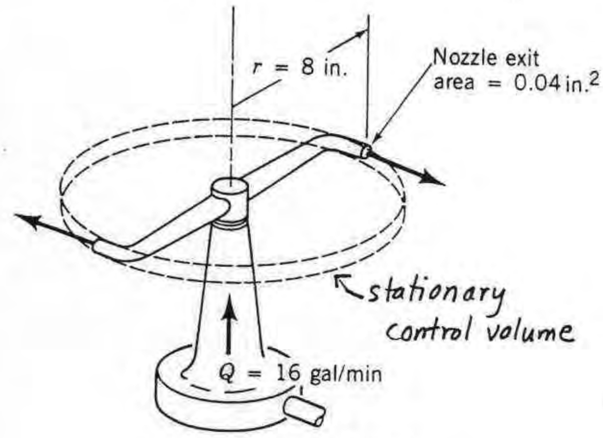


FIGURE P5.53

This is similar to Example 5.10.

(a) To determine the resisting torque required to hold the sprinkler head stationary we use the moment-of-momentum torque equation (Eq. 5.25). Thus,

$$T_{shaft} = \dot{m} r_2 V_{\theta,2} = \rho Q r_2 V_{\theta,2} \qquad (1)$$

For $V_{\theta,2}$ we use

$$V_{\theta,2} = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\ \frac{gal}{min}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{2\left(0.04\ in.^2\right)\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)}$$

or

$$V_{\theta,2} = 64.17\ \frac{ft}{s}$$

With Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\ \frac{slugs}{ft^3}\right)\left(16\ \frac{gal}{min}\right)\left(8\ in.\right)\left(64.17\ \frac{ft}{s}\right)\left(1\ \frac{lb}{slug\cdot\frac{ft}{s^2}}\right)}{\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)\left(12\ \frac{in.}{ft}\right)}$$

and

$$T_{shaft} = \underline{\underline{2.96\ ft\cdot lb}}$$

(b) To determine the resisting torque associated with a sprinkler speed of 500 $\frac{rev}{min}$ we use Eq. 1 again. However, with rotation we have

$$V_{\theta,2} = W_2 - U_2 \qquad (2)$$

For W_2 we use

$$W_2 = \frac{Q}{2A_{\substack{nozzle\\exit}}} = \frac{\left(16\ \frac{gal}{min}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{(2)\left(0.04\ in.^2\right)\left(7.48\ \frac{gal}{ft^3}\right)\left(60\ \frac{s}{min}\right)} = 64.17\ \frac{ft}{s}$$

(con't)

53 bi

5.53 (con't)

For U_2 we use

$$U_2 = r_2\omega = \frac{(8\text{ in.})\left(500\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)}{\left(12\ \frac{\text{in.}}{\text{ft}}\right)\left(60\ \frac{\text{s}}{\text{min}}\right)} = 34.91\ \frac{\text{ft}}{\text{s}}$$

Thus with Eq. 2 we have

$$V_{\theta,2} = 64.17\ \frac{\text{ft}}{\text{s}} - 34.91\ \frac{\text{ft}}{\text{s}} = 29.26\ \frac{\text{ft}}{\text{s}}$$

and with Eq. 1 we obtain

$$T_{shaft} = \frac{\left(1.94\ \frac{\text{slugs}}{\text{ft}^3}\right)\left(16\ \frac{\text{gal}}{\text{min}}\right)(8\text{ in.})\left(29.26\ \frac{\text{ft}}{\text{s}}\right)}{\left(7.48\ \frac{\text{gal}}{\text{ft}^3}\right)\left(60\ \frac{\text{s}}{\text{min}}\right)\left(12\ \frac{\text{in.}}{\text{ft}}\right)}\left(1\ \frac{\text{lb}}{\text{slug}\cdot\frac{\text{ft}}{\text{s}^2}}\right)$$

and

$$T_{shaft} = \underline{\underline{1.35\ \text{ft}\cdot\text{lb}}}$$

(c) To determine the angular velocity of the sprinkler if no resisting torque is applied we use the combination of Eqs. 1 and 2 to obtain

$$U_2 = W_2$$

or

$$\omega = \frac{W_2}{r_2} = \frac{\left(64.17\ \frac{\text{ft}}{\text{s}}\right)\left(12\ \frac{\text{in.}}{\text{ft}}\right)}{(8\text{ in.})} = 96.3\ \frac{\text{rad}}{\text{s}}$$

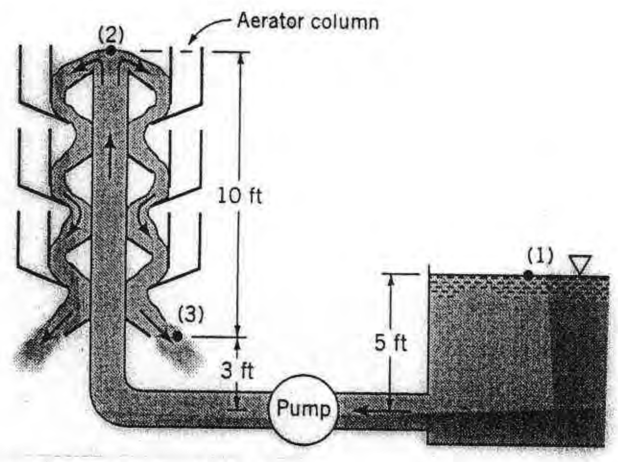
The rotor speed, N, is thus

$$N = \left(96.3\ \frac{\text{rad}}{\text{s}}\right)\frac{\left(60\ \frac{\text{s}}{\text{min}}\right)}{\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)} = \underline{\underline{920\ \frac{\text{rev}}{\text{min}}}}$$

53 bi cont

5.80

5.80 Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in **Video V5.14** and Fig. P5.80 at a rate of 3.0 ft^3/s. **(a)** Determine the power that the pump adds to the water if the head loss from (1) to (2) where $V_2 = 0$ is 4 ft. **(b)** Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is $V_3 = 2$ ft/s.



■ **FIGURE P5.80**

(a) The energy equation from (1) to (2)

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p - h_L = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

with

$p_1 = p_2 = V_1 = V_2 = 0$ gives

$$h_p = h_L + z_2 - z_1 = 4\,ft + (10+3)\,ft - 5\,ft = 12\,ft$$

Thus, the pump power is

$$\dot{W}_s = \gamma Q h_s = 62.4\,\frac{lb}{ft^3}\left(3\,\frac{ft^3}{s}\right)(12\,ft) = 2246\,\frac{ft\cdot lb}{s}\left(\frac{1\,hp}{550\,\frac{ft\cdot lb}{s}}\right)$$

$$= \underline{\underline{4.08\,hp}}$$

(b) The energy equation from (2) to (3)

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_p - h_L = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3$$

with

$p_2 = p_3 = V_2 = h_p = 0$ gives

$$h_L = z_2 - z_3 - \frac{V_3^2}{2g} = 13\,ft - 3\,ft - \frac{\left(2\,\frac{ft}{s}\right)^2}{2\left(32.2\,\frac{ft}{s^2}\right)} = 10\,ft - 0.062\,ft$$

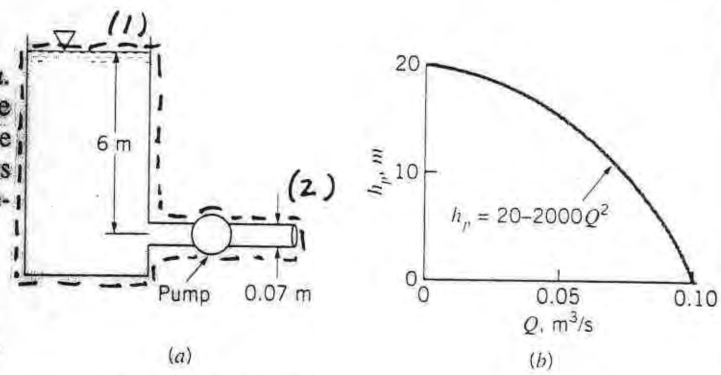
or

$$h_L = \underline{\underline{9.94\,ft}}$$

80br

5.82

5.82 Water is pumped from the tank shown in Fig. P5.82*a*. The head loss is known to be $1.2\,V^2/2g$, where V is the average velocity in the pipe. According to the pump manufacturer, the relationship between the pump head and the flowrate is as shown in Fig. P5.82*b*: $h_p = 20 - 2000\,Q^2$, where h_p is in meters and Q is in m³/s. Determine the flowrate, Q.



■ **FIGURE P5.82**

We want to know the flowrate Q.

For the control volume shown, application of the energy equation (Eq. 5.57) yields:

$$\cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\gamma}} + \overset{0}{\cancel{\frac{V_1^2}{2g}}} + z_1 + h_s - h_\ell \qquad (1)$$

However

$$h_\ell = 1.2\,\frac{V_2^2}{2g} \qquad (2)$$

and

$$h_s = h_p = 20 - 2000\,Q^2 \qquad (3)$$

Since $Q = V_2 A_2$ we have from eq. 2

$$h_\ell = \frac{1.2}{2g}\left(\frac{Q}{A}\right)^2 \qquad (4)$$

and combining Eqs. (1), (3) and (4) we get:

$$\frac{1}{2g}\left(\frac{Q}{A_2}\right)^2 + z_2 = z_1 + 20 - 2000\,Q^2 - \frac{1.2}{2g}\left(\frac{Q}{A_2}\right)^2 \qquad (5)$$

or

$$Q^2\left(\frac{1}{2gA_2^2} + \frac{1.2}{2gA_2^2} + 2000\right) = z_1 - z_2 + 20$$

So

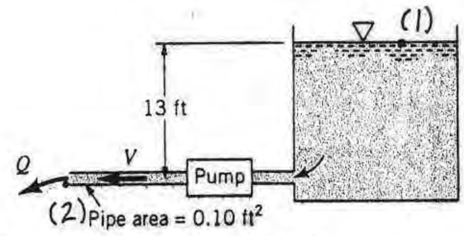
$$Q = \left[\frac{z_1 - z_2 + 20}{\frac{1}{2g\left(\frac{\pi d_2^2}{4}\right)^2} + \frac{1.2}{2g\left(\frac{\pi d_2^2}{4}\right)^2} + 2000}\right]^{\frac{1}{2}} = \left[\frac{6\,\text{m} + 20\,\text{m}}{\frac{1}{(2)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left[\frac{\pi(0.07\,\text{m})^2}{4}\right]^2} + \frac{1.2}{2\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)\left[\frac{\pi(0.07\,\text{m})^2}{4}\right]^2} + 2000}\right]^{\frac{1}{2}}$$

$$Q = 0.052\,\frac{\text{m}^3}{\text{s}}$$

82 br

5.84

5.84 Water is pumped from the large tank shown in Fig. P5.84. The head loss is known to be equal to $4V^2/2g$ and the pump head is $h_p = 20 - 4Q^2$, where h_p is in ft when Q is in ft^3/s. Determine the flowrate.



■ FIGURE P5.84

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g},$$ where $p_1 = p_2 = 0$, $z_1 = 13\,ft$, $z_2 = 0$, $h_s = h_p$ and $V_1 = 0$.

Thus,

(1) $$z_1 + h_p - h_L = \frac{V_2^2}{2g}$$

Also,

$$h_L = 4\frac{V^2}{2g} = 4\frac{V_2^2}{2g} = 4\frac{(Q/A_2)^2}{2g}$$ since $V_2 = \frac{Q}{A_2}$

Hence, Eq. (1) becomes

$$z_1 + (20 - 4Q^2) - 4\frac{(Q/A_2)^2}{2g} = \frac{(Q/A_2)^2}{2g}$$

or

$$\left[\left(\frac{5}{2g A_2^2}\right) + 4\right] Q^2 = 20 + z_1$$, where $g \sim \frac{ft}{s^2}$, $A_2 \sim ft^2$, and $Q \sim \frac{ft^3}{s}$

Thus, with the given data

$$\left[\left(\frac{5}{2(32.2\,\frac{ft}{s^2})(0.1\,ft^2)^2}\right) + 4\right] Q^2 = 20 + 13\,ft$$

or

$$\underline{\underline{Q = 1.67\,\frac{ft^3}{s}}}$$

84bi