Groundwater Homework 6

Question 1 - Consider a one-dimensional multiphase system with 2 fluid components that has arisen after a spill. One fluid is water. The other fluid is corn syrup. Assume that water is the wetting fluid. Assume a constant capillary pressure (Buckley-Leverett approximation). You know that the relative permeabilities are given by

$$k_{r1} = S^n \qquad k_{r2} = (1 - S)^n \quad \text{where} \qquad \text{n=2.5}$$

The intrinsic permeability of the medium is $1x10^{-8}$ m^2. The pressure gradient in fluid 1 is -15 Pa/m. What is the pressure gradient in fluid 2? Explain. For saturations S=0, 0.25, 0.5, 0.75 and 1 calculate the flow speeds of each phase. Compare and discuss. For each saturation which of the two phases has a greater volume flow rate?

Question 2 - Consider a 2-phase system of water and air in a 1-d vertical soil column. The medium has an intrinsic permeability of 100 milliDarcys (if you don't know what a millidarcy is look it up – commonly used by petroleum industry instead of other units). Assume air pressure is always and everywhere the same as atmospheric pressure. Briefly discuss this assumption and state if and when you think it is reasonable. Assume capillary pressure-saturation relationship

$$p_c = S_w^{-n} \quad \text{n=4.5}$$

At a given point it is measured that saturation varies as follows with depth.

Depth (m)	Saturation
0	0.1
4	1

You can interpolate linearly between the above points. Plot how S varies with depth. Now plot how capillary pressure varies with depth? Is there a flow of water and air in this system? Assume a quadratic relative permeability saturation relationship $k_{r1} = S^2$ and $k_{r2} = (1 - S)^2$ and calculate the Darcy velocity at 1 and 3 m? Is it up or down? Does it vary in space? If it varies in space, does this not violate continuity (conservation of mass), which says that in a 1-d system the flow speed is constant in space? Why or why not?

Question 3 – (This problem is not straightforward and you have to have really understood the last part on multiphase flow to do it as there are many subtleties in here…)

You are studying steady state, horizontal wetting in a laboratory column with no sources of water or air in the column. Assume you can use the Richards equation.

Write down the full Richards equation in both the tension and saturation based form. Next discard those terms from each of these equations, which you think can be discarded. Justify each term you discard. Finally, write the final simplified equation for both the tension and saturation based form.

Assume the Gardner model to relate saturation, relative permeability and suction. The Gardner model states

$K = K_s e^{\alpha\psi}$ and $\theta = e^{\alpha\psi}$

Using this model, which of the two forms of the Richards equation would you use to solve for the saturation profile in the column and why?

Take $K_s=1$ and $\alpha=3$. The porosity of the medium is 0.3. The column is 10 cm long. At x=0 the saturation is S=0 and at x=10cm the saturation S=1. Calculate the saturation profile in the column.

What changes in the saturation distribution if K_s were two orders of magnitude larger or smaller? Or if α were and order of magnitude larger or smaller?