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Chapter 3

Example Problem

$$\left.\begin{aligned} H &= 0.5\,m \\ W &= 5\,m \\ S &= 1/1000 \\ h_o &= 5\,cm = 0.05\,m \end{aligned}\right\} \longrightarrow u_* = \sqrt{gSH} = \sqrt{10\left(\tfrac{1}{1000}\right)0.5} = 0.07\,m/s$$

$$u_{avg} = \frac{u_*\left(H\ln\left(\frac{H}{h_o}\right) - H + h_o\right)}{\kappa H}$$

$$= \frac{0.07\left(0.5\left(\ln\left(\frac{0.5}{0.05}\right)\right) - 0.5 + 0.05\right)}{(0.4)(0.5)}$$

$$= 0.2455\ m/s$$

$$D_L = \frac{0.011\,(0.2455)^2\,(5)^2}{(0.07)(0.5)} = 0.4736\ m^2/s \qquad D_L = \frac{0.011\,u_{avg}^2\,W^2}{u_* H}$$

$$D_z = 0.067\,u_* H = (0.067)(0.2455)(0.5) = 0.0023\ m^2/s$$

$$D_y = 0.15\,u_* H = (0.15)(0.07)(0.5) = 0.0053\ m^2/s$$

$$\left.\begin{aligned} \tau_z &= H^2/D_z = (0.5)^2/0.023 = 10.87\,s \\ \tau_y &= W^2/D_y = 5^2/0.0053 = 4717\,s \end{aligned}\right\} \max(\tau_z, \tau_y) = 4717\,s$$

$$X_{crit} = \max(\tau_y, \tau_z)\,u_{avg} = 4717\,(0.2455) = 1158\,m$$

$$C = \frac{M}{(4\pi D_L t)^{1/2}}\,e^{-\frac{(x - u_{avg} t)^2}{4 D_L t}} = \frac{1}{(4\pi\,(0.4736)\,t)^{1/2}}\,e^{-\frac{(2316 - 0.2455t)^2}{4 D_L t}}$$

Consider This

$u = 1.2\ m/s$

$D_L = 0.1\ m^2/s$

$$C_{conservative} = \frac{1}{(4\pi D_L t)^{1/2}} e^{-\frac{(x-ut)^2}{4Dt}}$$

$$C_{reactive} = \frac{1}{(4\pi D_L t)^{1/2}} e^{-\frac{(x-ut)^2}{4Dt}} e^{-\gamma t}$$

$$\frac{C_{reactive}}{C_{conservative}} = e^{-\gamma t} = 0.2$$

Rearrange $\Rightarrow \gamma = \frac{-1}{t} \ln(0.2) = \frac{-1}{150} \ln(0.2)$

$$\gamma = 0.0107\ s^{-1}$$

Example

Steady State First Order Reactions

$$C = C_o \, e^{-\left(\frac{-v + \sqrt{v^2 + 4\gamma D}}{2D}\right)x}$$

$v = 0.4 \, m/s$
$D = 0.05 \, m^2/s$
$C_o = 3 \, g/l$
$\gamma = 0.001 \, s^{-1}$

$$\therefore C = 3 \, e^{-\left(\frac{-0.4 + \sqrt{0.4^2 + 4(0.001)(0.05)}}{2(0.05)}\right)(500)}$$

$$= 3 \, e^{-(2.5 \times 10^{-3})(500)}$$

$$= 0.86 \, g/l \Rightarrow \text{WAR!}$$

Sample Problem

Case 1 - Zeroth Order Reaction

$$D \frac{\partial^2 C}{\partial z^2} = \alpha e^{-\lambda z}$$

$C(z=H) = C_0$ $\quad$ $z = H$ is stream bed

$\left.\frac{\partial C}{\partial z}\right|_{z=0} = 0$ $\quad$ no flux at air-water interface

Solution $\Rightarrow$ $C = C_0 + \frac{\alpha}{D\lambda}\left(z + \frac{1}{\lambda} e^{-\lambda z}\right) - \frac{\alpha}{D\lambda}\left(H + \frac{1}{\lambda} e^{-\lambda H}\right)$

$C_0 = 1\,g/m$
$\alpha = 3\,g\,m^{-1}s^{-1}$
$\lambda = 20\,m^{-1}$
$D = 0.1\,m^2/s$

Question $\Rightarrow$ $r = ?$ $\Rightarrow$ $r = \alpha e^{-\lambda z} =$

Total reaction over depth

$$R = \int_0^H r\,dz = \int_0^{0.1} 3 e^{-20z}\,dz = \frac{-3}{20} e^{-20z}\Big|_0^{0.1}$$

$$= \frac{3}{20}\left(1 - e^{-2}\right) = 0.13\,g/s$$

Per day $\Rightarrow$ $0.13\,g/s \times 3600 \times 24 = 11.23\,kg$

Case 2 $\rightarrow$ BIG MESS $\rightarrow$ SEE MATHEMATICA

Lake Example

$V_1 = 500\ m^3$ $\quad V_2 = 250\ m^3$ $\quad V_3 = 250\ m^3$

$Q_{sw} = 20\ m^3 hr^{-1}$ $\quad Q_{GW_2} = 2\ m^2 hr^{-1}$ $\quad Q_{GW3} = 2\ m^2\ hr^{-1}$

$C_{sw} = 5\ g/l$ $\quad C_{GW} = 0$ $\quad C_{GW} = 0$

$Q_{1,2} = 5\ m^3/hr$ $\quad Q_{2,3} = 1\ m^3/hr$

Mass Balance

Layer 1 $\Rightarrow$

$$\cancel{500}\, V_1 \frac{dC_1}{dt} = Q_{sw} C_{sw} + Q_{GW_1} C_{GW} - (Q_{sw} + Q_{GW_1}) C_1 + Q_{1,2} (C_2 - C_1)$$

$$500 \frac{dC_1}{dt} = 20(5) + 0 - 20C_1 + 5C_2 - 5C_1$$

$$\Rightarrow \boxed{\frac{dC_1}{dt} = \frac{100 - 25C_1 + 5C_2}{500}}$$

Layer 2

$$V_2 \frac{dC_2}{dt} = Q_{GW_2} (C_{GW} - C_2) + Q_{1,2} (C_1 - C_2) + Q_{2,3} (C_3 - C_2)$$

$$250 \frac{dC_2}{dt} = 2(0 - C_2) + 5(C_1 - C_2) + 1(C_3 - C_2)$$

$$\boxed{\frac{dC_2}{dt} = \frac{5C_1 - 8C_2 + C_3}{250}}$$

Layer 3

$$V_3 \frac{dC_3}{dt} = Q_{GW_3} (C_{GW} - C_3) + Q_{2,3} (C_2 - C_3)$$

$$250 = 2 (0 - C_3) + 1 (C_2 - C_3)$$

$$\boxed{\frac{dC_3}{dt} = \frac{C_2 - 3C_3}{250}}$$

Steady State $\Rightarrow$

$$100 - 25C_1 + 5C_2 = 0 \qquad (\mathrm{I}) \Rightarrow 5C_1 = 20 + C_2$$

$$5C_1 \cancel{4} - 8C_2 + C_3 = 0 \qquad (\mathrm{II})$$

$$C_2 - 3C_3 = 0 \qquad (\mathrm{III})$$

$$C_3 = \tfrac{1}{3} C_2$$

$$(\mathrm{I} \rightarrow \mathrm{II}) \qquad 20 + C_2 - 8C_2 + \tfrac{1}{3} C_2 = 0$$

$$20 = \tfrac{20}{3} C_2$$

$$C_2 = 3 g/l$$

$$C_3 = 1 g/l$$

$$C_1 = 23/5 \ g/l$$

See Matlab for Rest