Question

$H = 2.5\,m$

$r = 0.25 - 4\ \mu m = 0.25 - 4 \times 10^{-6}\ m$

$C_o = 1 g\, m^{-3}$

$D_z = 0.1\, m^2/s$

Step 1 $\quad v_s = 3.33 \times 10^6 r^2 \rightarrow 2.1 \times 10^{-7}\, m/s$ (a) for $0.25\,\mu m$

$\rightarrow 5.3 \times 10^{-5}\, m/s$ (b) $4\,\mu m$

Let's start with $C = C_o e^{-v_s/D\, z}$

$$\langle C \rangle = \frac{1}{H}\int_0^H C_o e^{-\frac{v_s}{D} z}\, dz$$

average

$$= -\frac{C_o D}{H v_s}\left[e^{-\frac{v_s}{D} z}\right]_0^H$$

$$= \frac{C_o D}{H v_s}\left(1 - e^{-\frac{v_s H}{D}}\right)$$

(a) $\langle C \rangle = 0.999\ g/m^3$

(b) $\langle C \rangle = 0.999\ g/m^3$

Settling velocity is so low that particles are suspended throughout

$\langle C \rangle_{toxin} = 0.12 \langle C \rangle \approx 0.12\ g/m^3$

For this particular problem $C = \langle C \rangle$ ~ i.e. there is not much vertical variability

$\Rightarrow$ averages are fine

But in general NO!

Example

Current conditions $\quad C_{1a} \quad C_{1w}$

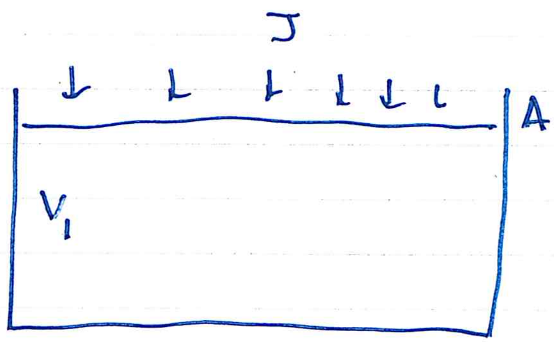
Polluted conditions $\quad C_{2a} \quad 2_{2w}$

At equilibrium $\left.\begin{aligned} C_{1w} &= \alpha C_{1a} \\ C_{2w} &= \alpha C_{2a} \end{aligned}\right\}$ If $C_{2a} = 2C_{1a}$

$\Rightarrow C_{2w} = 2C_{1w}$

$\Downarrow$

Fish done for



Mass Balance $\Rightarrow V \dfrac{dC_w}{dt} = JA = -kA\,(C_w - \alpha C_a)$

Assume $C_a = C_{2a}$

$$\Rightarrow \frac{dC_w}{dt} = \frac{-kA}{V}\,(C_w - \alpha C_a)$$

$$C_w(t=0) = C_{1w}$$

Solution
(Mathematica)

$$C_w = \alpha C_{a2} + (C_{1w} - \alpha C_{a2})\, e^{-\frac{kA}{V}t}$$

Now when is $C_w = 1.5\, C_{1w}$

$$= 0.75\, C_{2w}$$

$$= 0.75\, \alpha C_{2a} \quad ?$$

Likewise $\quad C_{1w} = \alpha C_{1a} = 0.5\, \alpha C_{2a}$

$$\therefore \quad 0.75\, \alpha C_{2a} = \alpha C_{2a} + \left(0.5 \alpha C_{2a} - \alpha C_{2a}\right) e^{-\frac{kA}{V} t}$$

$$0.75 = 1 + (0.5 - 1)\, e^{-\frac{kA}{V} t}$$

$$\frac{-0.25}{-0.5} = \frac{1}{2} = e^{-\frac{kA}{V} t}$$

$$\Rightarrow \quad t = \frac{V}{kA} \ln(2)$$

$$t_1 = \frac{5000}{(10^{-8})(500)} \ln(2) = 6.93 \times 10^{8}\, s \approx 21 \text{ years}$$

$$t_2 = \frac{5000}{(10^{-3})(500)} \ln(2) = 6.9 \times 10^{3}\, s \approx 2 \text{ hours}$$