Sample Problem

(i) $\frac{dN}{dt} = \alpha N \Rightarrow N = N_0 e^{\alpha t}$

Time	Biomass (mg/L)	ΔN (mg/L)	Δt (d)	$\Delta N/\Delta t$	Avg N	α (day^{-1})
0	50					
1	68	18	1	18	59	0.3
2	91	23	1	23	70	0.3
3	123	32	1	32	107	0.3
4	166	43	1	43	147	0.3
5	224	58	1	58	195	0.3

$$\alpha = 0.3\ day^{-1}$$

(ii) $N = N_0 e^{\alpha t}$

$= 50\ e^{(0.3)(25)} = 9 \times 10^4$ mg/L

(iii) $N = \frac{K}{1 + \left(\frac{K}{N_0} - 1\right) e^{-\alpha t}} = \frac{50000}{1 + \left(\frac{50000}{50} - 1\right) e^{-0.3(25)}} = 32205$ mg/L

$\frac{32205}{90000} \Rightarrow 35.6\%$ of exponential growth model

(iv) $0.3\,d^{-1} = \alpha\left(\frac{S}{G+S}\right) = \alpha\left(\frac{25}{5+25}\right) = 0.83\,\alpha$

$\Rightarrow \alpha = \frac{0.3}{0.83} = 0.36\ day^{-1}$

$\Rightarrow$ 120% faster

(v) Initial Growth Rates

$\mu = \alpha\left(\frac{S}{G+S}\right) \Rightarrow \mu_A = 1\left(\frac{100}{100+0.1}\right) = 1\,d^{-1}$

$\mu_B = 5\left(\frac{100}{100+5}\right) = 4.76\,d^{-1}$

$\Rightarrow$ B more effective at these concentrations

Once $S \approx 1$ mg/L

$\Rightarrow \mu_A = 1\left(\frac{1}{1+0.1}\right) = 0.9\,d^{-1}$

$\mu_B = 5\left(\frac{1}{1+5}\right) = 0.8\,d^{-1}$

$\Rightarrow$ A better than B for lower concentrations

(vi) $\frac{dN}{dt} = -k_d N \quad \Rightarrow \quad N = N_0 e^{-k_d t}$

$$t = \frac{-1}{k_d} \ln\left(\frac{N}{N_0}\right) = \frac{-1}{0.35} \ln\left(\frac{15}{90}\right) = 5.1 \text{ days}$$