Example - Gramling's Experiment

$$Da = \frac{k C_0 D}{u^2} = \frac{k C_0 \alpha \kappa}{\kappa^2} = \frac{k C_0 \alpha}{\kappa}$$

$$= \frac{(4.4 \times 10^5)(1)(10^{-3})}{\left(\frac{.1 \times 10^{-2}}{0.4}\right)}$$

$$= 17600 \gg 100$$

$\Rightarrow$ Instantaneous

Assume $-\infty < x < \infty$

$CuSO_4$ is labeled C_A $\qquad C_A(t=0) = H(-x)$

EDTA " " C_B $\qquad C_B(t=0) = H(x)$

step functions separated by sharp interface at $x = 0$

$C_C(t=0) = 0$

Define $u_A = C_A + C_C \Rightarrow u_A(t=0) = H(-x)$

$u_B = C_B + C_C \Rightarrow u_B(t=0) = H(x)$

u_A, u_B are conservative

$$\Rightarrow \frac{\delta u_A}{\delta t} + v \frac{\delta u_A}{\delta x} = D \frac{\delta^2 u_A}{\delta x^2}$$

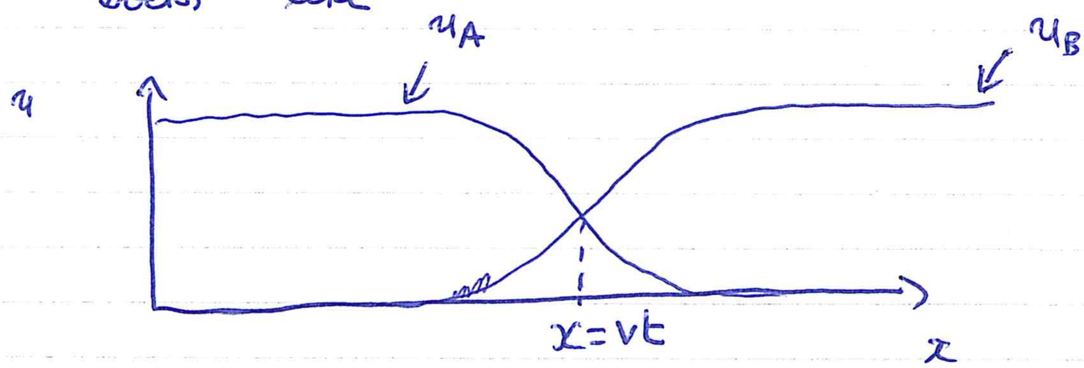
For this initial condition

$$\Rightarrow$$

$$u_A = \frac{1}{2} \operatorname{erfc}\left(\frac{x - vt}{\sqrt{4Dt}}\right)$$ ~ See chapter 1

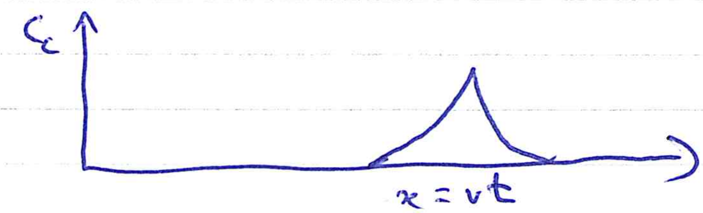
Similarly $$u_B = \frac{1}{2} \operatorname{erfc}\left(\frac{vt - x}{\sqrt{4Dt}}\right)$$

Looks like



Recall $C_c = \min(u_A, u_B)$

$\Rightarrow$



$$\therefore \quad c_C = \begin{cases} u_B & x < vt \\ u_A & x > vt \end{cases}$$

$$M_C = \int_{-\infty}^{\infty} c_C \, dx$$

Total mass of product

$$= 2\int_{vt}^{\infty} u_A \, dx = 2\int_{-\infty}^{vt} u_B \, dx$$

$$= 2\int_{vt}^{\infty} \frac{1}{2} \operatorname{erfc}\left(\frac{x - vt}{\sqrt{4Dt}}\right) dx$$

$$= 2\sqrt{\frac{Dt}{\pi}}$$

Does not depend on v

Scales like $\sqrt{t} = t^{1/2}$ $\rightsquigarrow$ width of mixing zone