Problem 1. Analytical Differential Equations

Solve the following differential equations using any means - Mathematica may be the most straightforward. Do not Worry if solutions are written in terms of functions that you do not recognize - e.g. Bessel functions. These are well documented and most software and good calculators can calculate them for you.

(a)
$$\frac{\partial^2 C}{\partial x^2} = -\alpha C$$

(b)
$$\frac{\partial}{\partial x}\left(e^{\beta x}\frac{\partial C}{\partial x}\right) = \alpha C$$

(c)
$$\frac{\partial^2 C}{\partial x^2} - 10\frac{\partial C}{\partial x} = 2C \qquad C(0) = 1; C(10) = 0;$$

(d)
$$\frac{\partial}{\partial x}\left(3x\frac{\partial C}{\partial x}\right) = 12x \qquad C'(1) = 0; C(5) = 2;$$

For (a) and (b) write the solutions in general form with the constants of integration. For (c) and (d), calculate results explicitly and plot them. Look at the results and see if they make sense to you in terms of what the differential equation is telling you.

Problem 2. Numerical Differential Equations

This problem is to get you comfortable with finite difference numerical approach. You can code in any language you like, including Excel, but I recommend Matlab

(a) Run this problem out to $x = 10$. Make sure your step is small enough.

$$\frac{\partial C_1}{\partial x} = -2C_1 + 3C_2 - 1 \qquad \frac{\partial C_2}{\partial x} = 2C_1 - 3C_2 + 2 \qquad C_1(0) = 10 \qquad C_2(0) = 2$$

(b) Run this problem out to $t = 10$. Make sure your step is small enough.

$$\frac{\partial C_1}{\partial t} = -C_1 + C_2 \qquad \frac{\partial C_2}{\partial t} = C_1 - 3C_2 \qquad \frac{\partial C_3}{\partial t} = 2C_2 - 4C_3 \qquad \frac{\partial C_4}{\partial t} = 4C_3 - 3C_4$$
$$C_1(0) = 10 \qquad C_2(0) = 0 \qquad C_3(0) = 0 \qquad C_4(0) = 0$$

Plot the solutions and discuss them. Do they make sense to you? Is what you get for large x or large t consistent with what you expect?