Problem 1. Control Volumes

A Lake is $10^4 m^3$ in size and is fed by a river and two streams and drained by one large river. In general it is known that the volume of the river does not change much, if at all. The two small streams have flow rates of 45 and 65 $m^3/hour$ respectively. The large river draining the lake has a flow rate of 240 $m^3/hour$.

- Stream 1 is pristine
- Stream 2 is highly contaminated with a contaminant of concentration 10 $g/liter$
- The River flowing into the lake has a lower concentration of the same contaminant of 3 $g/liter$.

Assume that at time $t = 0$ the water in the lake is pristine. Estimate how the concentration in the river that drains the lake evolves over time. After a sufficiently long time the concentration in the lake will reach some maximum value. What is that value?

Now assume that a remediation effort is succesful and that both of the contaminated water bodies flowing into the lake are suddenly pristine. How long will it take before the lake becomes clean?

State any assumptions that you make very clearly. Feel free to make bold assumptions - I will reward them as what I am looking for here is your ability to assess a problem quickly and efficiently.

Problem 2. Pulse Advection Diffusion

Two point contaminant spills, one of mass $15kg$ and the other of mass $30kg$, occur in a channel that you can treat as a one-dimensional system. The $15kg$ spill occurs at location $x = 0$ and the other spill occurs at $x = 25m$. You can assume that the domain is very large relative to your area of interest. You measure the following properties of the flow :

- Flow velocity: 0.1 m/s
- Molecular Diffusion: 10^{-9} m^2/s
- Turbulent Dispersion: 10^{-2} m^2/s
- Mechanical Dispersion: 5×10^{-3} m^2/s

(a) Write down a mathematical expression for how the total concentration field evolves in space and time.

(b) Plot the evolution of concentration against time at location $x = 100m$ - i.e. plot the breakthrough curve there. Now do the same thing, but imagine that the turbulent and mechanical dispersion coefficients were ten times bigger than they are. Discuss the differences and commonalities between the two results. (*Hint: For each case, you want to plot the solution to spill 1 and the solution to spill 2 separately as well as the total solution - on the same plot*).

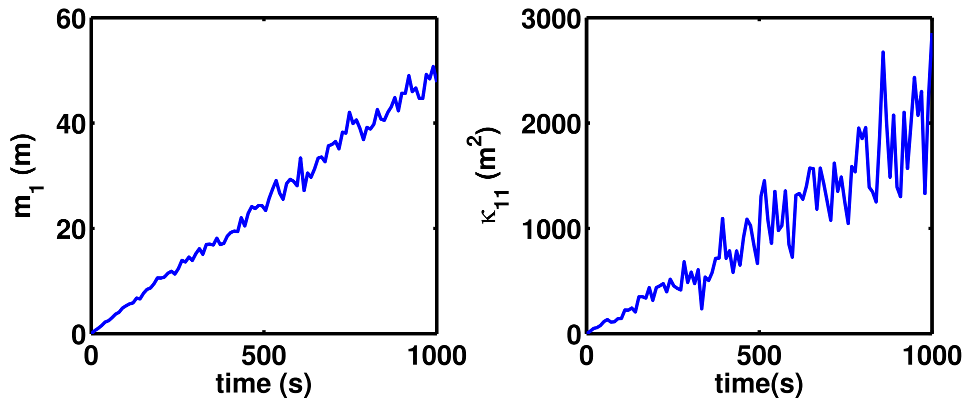

Figure 1: First and Second Centered Moments for Problem 3

(c) For the spill from x=0, write down an expression for how the location of the peak of the plume associated with it evolves in time, an expression for how the peak concentration evolves in time and an expression for how the characteristic width of the plume evolves in time. Now calculate each of these at a time of 1 hour.

Problem 3. Moments

You are provided with the graphs of spatial moments for the first and second centered moment of a plume depicted in Figure 1. Additionally you have measured a breakthrough curve at a location $x = 100m$ downstream of where the spill occurred. You know that the spill was of mass $1kg$. From your breakthrough curve you estimate that the zeroth temporal moment is $M_0 = 20kgs/m$ and that the first moment is $M_1 = 5.6 \times 10^5 kgs^2/m$. For both calculate the velocity and total dispersion coefficient. Are they comparable?

Problem 4. Non-Pulse Spill

A spill of uniform concentration $25g/l$ has occurred over a distance of $75m$. You may assume the system is one dimensional , the flow velocity is $0.25m/s$ and total dispersion coefficient including molecular, turbulent and mechanical components is $3 \times 10^{-2} m^2/s$.

(a) Write down the solution for the evolution of the concentration field

(b) Plot the breakthrough curve at a distance 200 meters down stream of the leading edge of the contamination. What is the maximum concentration to pass that point? How about at 2000 meters downstream?