Homework 3

① (i) $u_* = \sqrt{gHS} = \sqrt{(9.8)(0.75)\left(\frac{1}{2500}\right)} = 0.0542 \ m s^{-1}$

$D_y = 0.15\, u_* H = (0.15)(0.0542)(0.75) = 6.1 \times 10^{-3} m^2 s^{-1}$

$D_z = 0.067\, u_* H = (0.067)(0.0542)(0.25) = 2.724 \times 10^{-3} m^2 s^{-1}$

$\tau_y = \frac{W^2}{D_y} = \frac{4^2}{6.1\times10^{-3}} = 2624 \ s$ $\qquad \tau_z = \frac{H^2}{D_z} = \frac{(0.75)^2}{2.72\times10^{-3}} = 206.5 \ s$

$\tau_y > \tau_z = \tau = 2624 \ s$

$$u_{avg} = \frac{u_* \left(H \ln\left(\frac{H}{h_0}\right) - H + h_0 \right)}{KH} = \frac{(0.05)\left(0.75\left(\ln\left(\frac{0.75}{0.015}\right)\right) - 0.75 + 0.015\right)}{0.4\,(0.75)}$$

$= 0.4 \ m/s$

$l_{crit} = \tau u_{avg} = 2624\,(0.4) = 1050 \ m$

$450 \ m < l_{crit} \Rightarrow \boxed{\text{Not well mixed}}$

(ii) $$C = \frac{M}{(4\pi D_x t)^{1/2}} e^{-\frac{(x - u_{avg} t)^2}{4 D_x t}}$$

$D_x = \frac{0.011\, u_{avg}^2 W^2}{u_* H} = 0.68 \ m^2/s$

$M = C (4\pi D_x t)^{1/2} e^{+\frac{(x - vt)^2}{4 D_x t}}$

$$t = \frac{5000}{u_{avg}}$$

$$M = \frac{(100\times10^{-6})\left(4\pi(0.68)\left(\frac{5000}{0.4}\right)\right)^{1/2}}{e^{-\frac{(5000-5000)}{4D_x t}}} = \boxed{0.033\,g}$$

$$\sigma_{width} = \sqrt{2D_x t} = \sqrt{2(0.68)\left(\frac{5000}{0.4}\right)} \approx \boxed{130\,m}$$

② (i) $\gamma = 0.2\,hr^{-1} = 5.6\times10^{-5}\,s^{-1}$

$$C = \frac{M}{(4\pi D_x t)^{1/2}}\, e^{-\frac{(x-ut)^2}{4D_x t}}\, e^{-\gamma t}$$

$$C_{peak} = \frac{M}{(4\pi D_x t)^{1/2}}\, e^{-\gamma t} \qquad t = \frac{5000}{0.4} = 12500\,s$$

By design from ①

$$= 100\,\mu g\; e^{-(5.6\times10^{-5}\times12500)}$$

$$= 50\,\mu g/m$$

(ii) Steady State

$$C = C_o \, e^{\frac{u - \sqrt{u^2 + 4\gamma D_x}}{2D} x}$$

$u = 0.4 \quad ms^{-1}$

$\gamma = 5.6 \times 10^{-5} \; s^{-1}$

$D_x = 0.682 \; m^2 s^{-1}$

$x = 5000 \; m$

$C_o = 25 \; g/l$

$\therefore \; C = 12.4 \; g/l$ ~ consistent with (i) → i.e about half

(3) (i) $R = \int_0^H r \, dz = \int_0^H \alpha e^{-\lambda z} dz = -\frac{\alpha}{\lambda}\left(e^{-\lambda H} - 1\right)$

$$\therefore R = -\frac{0.1}{10}\left(e^{-(10)(2.5)} - 1\right) = 0.01 \, g \, min^{-1}$$

$$\boxed{= 14.4 \, g \, day^{-1}}$$

(ii) $C = C_o + \frac{\alpha}{D_z \lambda}\left(z + \frac{1}{\lambda} e^{-\lambda z}\right) - \frac{\alpha}{D_z \lambda}\left(H + \frac{1}{\lambda} e^{-\lambda H}\right)$

@ $z = 0 \Rightarrow C = 10 \, g/m + \frac{0.1}{D_z (10)}\left(0 + \frac{1}{10} e^{0}\right) - \frac{0.1}{D_z (10)}\left(2.5 + \frac{1}{10} e^{-10(2.5)}\right)$

Assume $D_z = 0.1 \, m^2/s$ $\Rightarrow$ $\boxed{C = 9.76 \, g/m}$

↳ forgot to specify

(iii) If sunlight doubles $\Rightarrow$ reaction rate $\propto$ doubles

(iv) Zeroth order reaction o.k. because C varies very little in the water column

$$9.76 \leq C \leq 10 \quad g/m$$

(4)

Layer 1 $$V_1 \frac{dC_1}{dt} = Q_{GW_1} \cancel{C_{GW_1}}^{0} + Q_{SW_1} C_{SW_1} + Q_{SW_2} \cancel{C_{SW_2}}^{0} + Q_{12}(C_2 - C_1) - Q_R C_1 - Q_{GW} C_1$$

$$8000 \frac{dC_1}{dt} = 10 C_{S1} + 3(C_2 - C_1) - 20 C_1 - 2C_1$$

$$\boxed{8000 \frac{dC_1}{dt} = 10 C_{S1} - 25 C_1 + 3 C_2} \quad (A)$$

Layer 2 $$V_2 \frac{dC_2}{dt} = Q_{GW_2} \cancel{C_{GW_2}}^{0} + Q_{12}(C_1 - C_2) + Q_{23}(C_3 - C_2) - Q_{GW} C_2$$

$$7000 \frac{dC_2}{dt} = 3(C_1 - C_2) + 0.5(C_3 - C_2) - 2 C_2$$

$$\boxed{7000 \frac{dC_2}{dt} = 3C_1 - 5.5 C_2 + 0.5 C_3} \quad (B)$$

Layer 3

$$V_3 \frac{dC_3}{dt} = Q_{GW_3} \cancel{C_{GW_{in}}} + Q_{23}(C_2 - C_3) - Q_{GW} C_3$$

$$5000 \frac{dC_3}{dt} = 0.5(C_2 - C_3) - 2C_3$$

$$\boxed{5000 \frac{dC_3}{dt} = 0.5C_2 - 2.5C_3} \quad (C)$$

Steady State $\Rightarrow \frac{dC_1}{dt} = \frac{dC_2}{dt} = \frac{dC_3}{dt} = 0$

$\therefore$ From (C) $\Rightarrow 0 = 0.5C_2 - 2.5C_3 \Rightarrow \boxed{C_2 = 5C_3}$

(B) $\Rightarrow 3C_1 - 5.5C_2 + 0.5C_3 = 0$

$$3C_1 = 5.5C_2 \mp 0.1C_2 = 5.6C_2$$

$$\boxed{C_1 = \frac{5.4}{3} C_2}$$

(A) $10\,C_{stream} - 25C_1 + 3C_2 = 0$

$$\boxed{C_{stream} = \frac{25C_1 - 3C_2}{10}}$$

$C_3 = 1\,g/l \Rightarrow C_2 = 5\,g/l \Rightarrow C_1 = 9\,g/l$

$$\Rightarrow \boxed{C_{stream} = 21\,g/l}$$

Final part numerical !