HW5

(1)

$$C(x,y,t) = \frac{M}{4\pi (D_x D_y)^{1/2} t/R} \; e^{-\frac{(x-\frac{ut}{R})^2}{4D_x t/R} - \frac{y^2}{4D_y t/R}}$$

where $M = 12\,kg$

$$u = \frac{15}{0.35} = 42.86 \text{ m/day}$$

$D_x = \alpha_L |u| = (0.02)\left(\frac{15}{0.35}\right) = 0.8571 \text{ m}^2/\text{day}$ (neglecting D_{mol})

$D_y = \alpha_T |u| = (0.002)\left(\frac{15}{0.35}\right) = 0.0857 \text{ m}^2/\text{day}$ (" ")

$R = 2$ or 4 depending

Now plot at $(x,y) = (5,5), (40,10), (100,10), (100,20)$

$R=2$ and 4 give the same results → just delayed for $R=4$

See plots attached → note tiny concentrations in all cases because of well locations

(2)

(i) $C = \frac{C_0}{4} e^{\frac{x}{2\alpha_x}\left(1-\sqrt{\frac{u+4\gamma\alpha_x}{u}}\right)} \operatorname{erfc}\left(\frac{x-ut\sqrt{\frac{u+4\gamma\alpha_x}{u}}}{2\sqrt{\alpha_x u t}}\right)\left[\operatorname{erf}\left(\frac{y+L/2}{2\sqrt{\alpha_y x}}\right)-\operatorname{erf}\left(\frac{y-L/2}{2\sqrt{\alpha_y x}}\right)\right]$

$C_0 = 15\ g/l$

$u = \frac{2}{0.15} = 13.3\ m/day$

$\alpha_x = 10^{-3}\ m$

$\alpha_y = 10^{-4}\ m$

$L = 5$

} Plug into formula above and plot for $(x,y) = (10,0), (10,-2), (10,2)$

Roughly $x = ut \Rightarrow 10 = 13.3t \Rightarrow t = 0.75$ days to read steady state

(ii) $C = 15/100\ g/l$

$\therefore C = \frac{C_0}{4} e^{\frac{x}{2\alpha_x}\left(1-\sqrt{\frac{u+4\gamma\alpha_x}{u}}\right)} \quad (2)\ (2)$

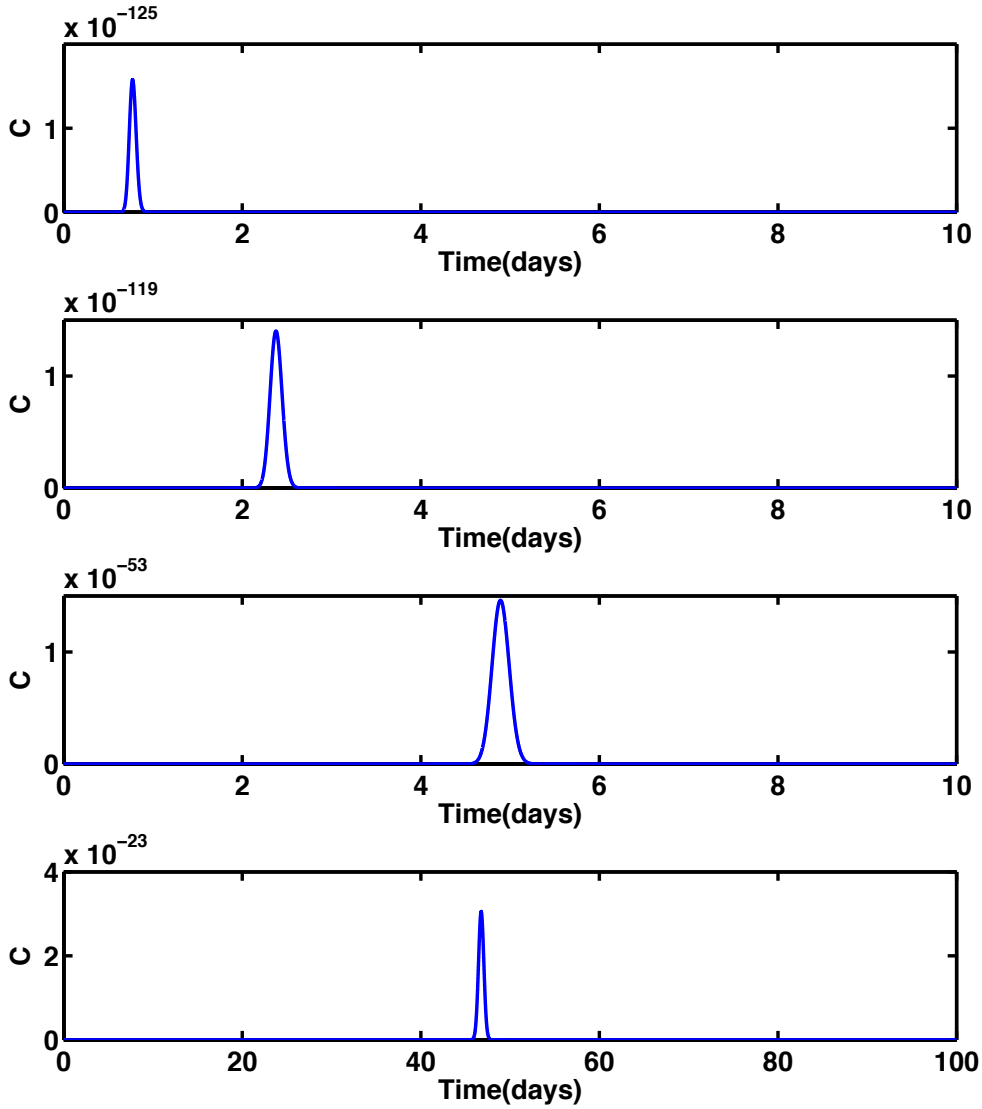
$\therefore \frac{15}{100} = \frac{15}{4} e^{\frac{10}{2\times10^{-3}}\left(1-\sqrt{\frac{13.3+4\gamma\ 10^{-3}}{13.3}}\right)} \quad (4)$

Solve for $\gamma \Rightarrow \gamma = 6.12\ day^{-1}$

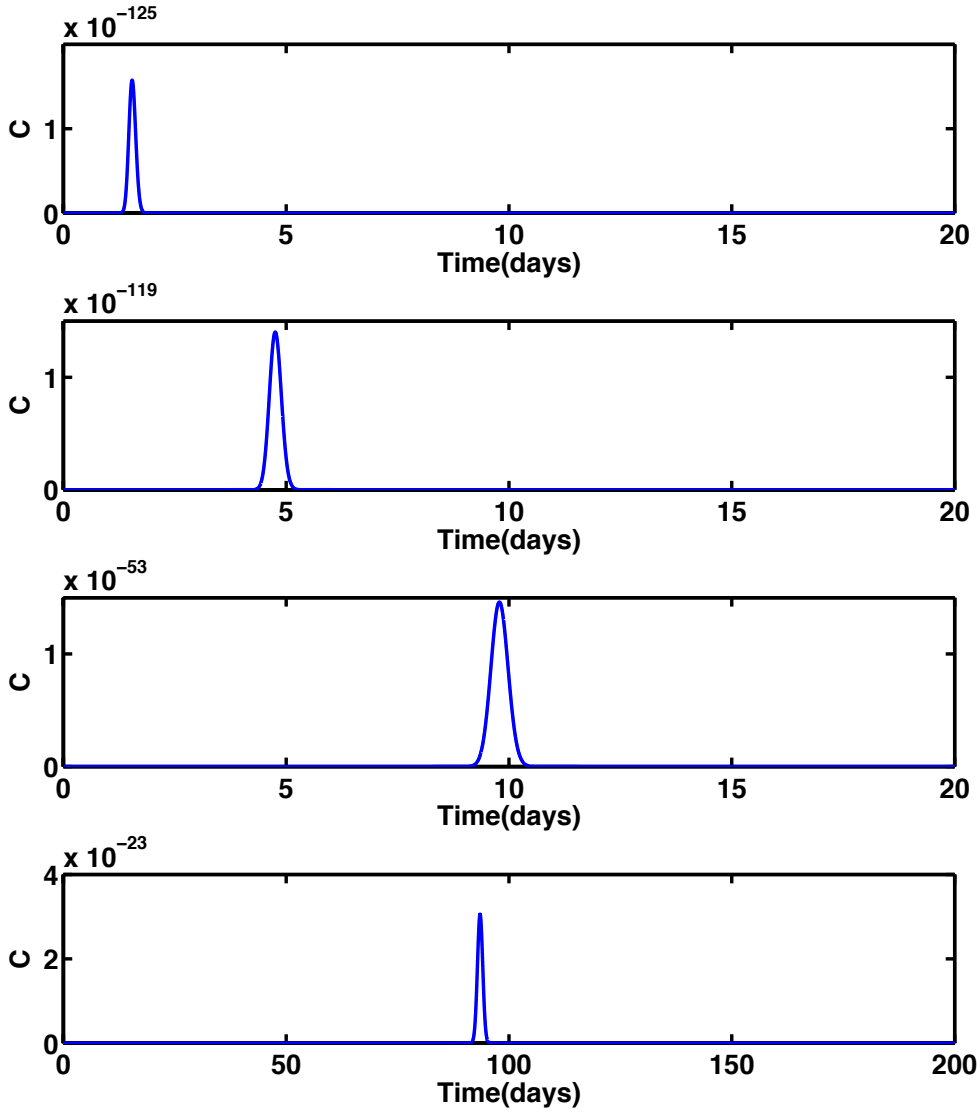
$$C(x,y) = \frac{15}{4} e^{\frac{x}{2\alpha x}\left(1-\sqrt{\frac{u+4\gamma\alpha x}{u}}\right)} (2)\left[\operatorname{erf}\left(\frac{y+4/2}{2\sqrt{\sigma_y x}}\right) - \operatorname{erf}\left(\frac{y-4/2}{2\sqrt{\sigma_y x}}\right)\right]$$

Plug in desired values and go

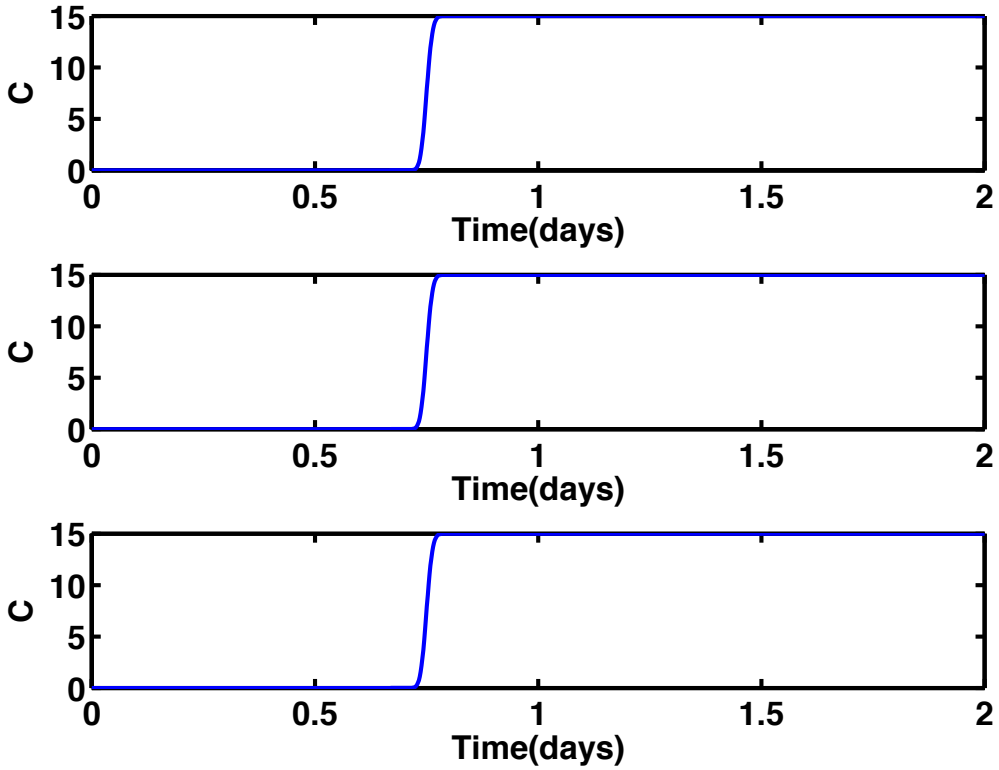
Problem 1 R=2


x 10^−125
C
1
0
0
2
4
6
8
10
Time(days)
x 10^−119
C
1
0
0
2
4
6
8
10
Time(days)
x 10^−53
C
1
0
0
2
4
6
8
10
Time(days)
x 10^−23
C
4
2
0
0
20
40
60
80
100
Time(days)

Problem 1 R=4 – notice same as R=2, just delayed (stretched x a-xis)



Problem 2


15
10
5
0
C
0
0.5
1
1.5
2
Time(days)
15
10
5
0
C
0
0.5
1
1.5
2
Time(days)
15
10
5
0
C
0
0.5
1
1.5
2
Time(days)

With gamma=6.12 day $^{-1}$