HW6

① (i) $N = N_o e^{\alpha t}$

From data best fit is $\alpha = 0.25\ day^{-1}$

Can check with $\alpha = \frac{1}{t} \ln\left(\frac{N}{N_o}\right) = \frac{1}{1} \ln\left(\frac{10.5850}{5}\right)$

$= \frac{1}{2} \ln\left(\frac{22.4084}{5}\right)$

(ii) Logistic Growth

$$\Rightarrow \quad N = \frac{K}{1 + \left(\frac{K}{N_o} - 1\right) e^{-\alpha t}}$$

$K = 10000$

$N_o = 5$

$\alpha = 0.25$

$$\Rightarrow \quad N = \frac{10000}{1 + \left(\frac{10000}{5} - 1\right) e^{-(0.25)(10)}}$$

$= 4749\ mg/l$

(iii) $N = N_o e^{-k_d t}$

After 1 day $N = 0.25\ N_o \Rightarrow k_d = -\frac{1}{\cancel{t}} \ln(0.75)$

$= 0.2877\ day^{-1}$

(iv) $$S = \frac{k_d G}{\alpha - k_d}$$

$$= \frac{(0.2877)(0.1)}{0.75 - 0.2877} = 0.0622 \, mg/l$$

(v) Either reduce G by factor of 10 or k_d

$\Rightarrow$ G better as you may want bacteria to die quickly post clean up!

(2) $$\frac{dC_A}{dt} = -k C_A C_B$$

$$\left.\begin{array}{l} C_A(t=0) = C_o \\ C_B(t=0) = C_o + \Delta C \end{array}\right\} \Rightarrow C_A(t) = C_B(t) - \Delta C$$ at all times

$$\frac{dC_A}{dt} = -k C_A (C_A + \Delta C)$$

$\Rightarrow$ Mathematica: or solve by any other means

$$\Rightarrow \text{(i)} \; C_A(t) = \frac{C_o \Delta C}{(C_o + \Delta C) e^{\Delta C k t} - C_o}$$

(ii) Exponential rather than power law decay $\Rightarrow$ faster

(iii) $C_0 = 1 \text{ mol m}^{-3}$
$\Delta C = 1 \text{ mol m}^{-3}$

$k = 0.1 \text{ m}^3 \text{s}^{-1} \text{mol}^{-1}$

$$\therefore C = \frac{(1)(1)}{2e^{0.1t} - 1}$$

When is $C = 0.01$ $\Rightarrow$ $0.01 = \frac{1}{2e^{0.1t} - 1}$

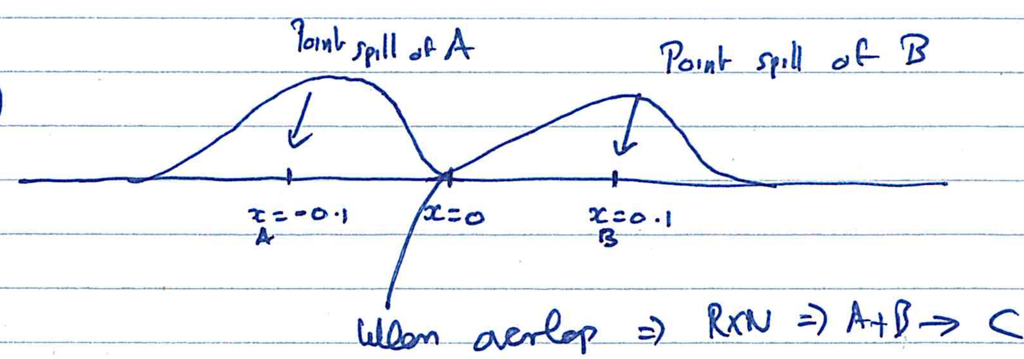
$\Rightarrow 2e^{0.1t} - 1 = 100$

$t = \frac{1}{0.1} \ln(55)$

$= 40.07 \text{s}$

③

(i)



(ii) $\frac{\delta C_A}{\delta t} = D \frac{\delta^2 C_A}{\delta x^2} - r$ $\qquad$ $\frac{\delta C_B}{\delta t} = D \frac{\delta^2 C_B}{\delta x^2} - r$

$\frac{\delta C_c}{\delta t} = D \frac{\delta^2 C_c}{\delta x^2} + r$

$C_A C_B = 0$

$$C_A(t=0) = \delta(x-x_A)$$

$$C_B(t=0) = \delta(x-x_B)$$

$$C_c(t=0) = 0$$

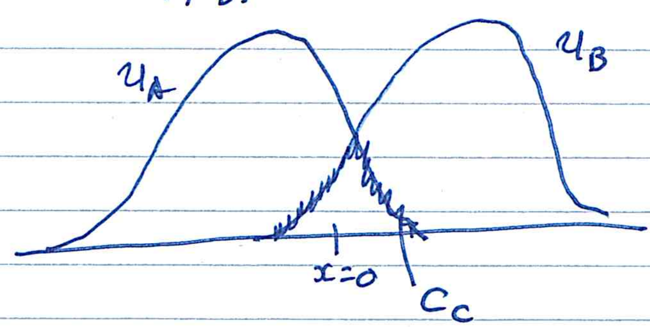
Define $u_A = C_A + C_c \Rightarrow u_A(t=0) = \delta(x-x_A)$

$$u_B = C_B + C_c \Rightarrow u_B(t=0) = \delta(x-x_B)$$

$$\frac{\delta u_A}{\delta t} = D\frac{d^2 u_A}{dx^2} \Rightarrow u_A(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-\frac{(x-x_A)^2}{4Dt}}$$

$$\frac{\delta u_B}{\delta t} = D\frac{d^2 u_B}{dx^2} \Rightarrow u_B(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-\frac{(x-x_B)^2}{4Dt}}$$

$$C_c = \min(u_A, u_B)$$



$$M_c = \int_{-\infty}^{\infty} c_c\,dx = 2\int_0^{\infty} c_c\,dx$$

$$= 2\int_0^{\infty} u_A\,dx$$

$$\text{OR } 2\int_{-\infty}^{0} u_B\,dx$$

$$M_c = 2 \int_0^{\infty} \frac{1}{\sqrt{4\pi D t}} \, e^{-\frac{(x - x_A)^2}{4Dt}} \, dx$$

$$= 1 + \mathrm{erf}\left(\frac{x_A}{\sqrt{4Dt}}\right)$$

$x_A = -0.1$ m

$D = 10^{-6}$ $m^2 s^{-1}$

$$\cancel{\Rightarrow M_c = 1 + \mathrm{erf}\left(\right.}$$

$$M_c = 1 - \mathrm{erf}\left(50 t^{-1/2}\right)$$

inverse error function

↓

erfinv in Matlab.

$$t = \left(\frac{1}{50} \mathrm{erf}^{-1}(1 - M_c)\right)^{-2}$$

When is $M_c =$

0.1	$\longrightarrow$	1.85×10^3 s
0.25		3.7×10^3 s
0.5		1.099×10^4 s
0.9		3.166×10^5 s
0.95		1.27×10^6 s