Problem 1

(a)

```
eqa := C''[x] == -alpha * C[x]
```

```
eqa
```

$C''[x] == -\text{alpha}\, C[x]$

```
DSolve[eqa, C, x]
```

$\{\{C \to \text{Function}[\{x\}, C[1] \text{Cos}[\sqrt{\text{alpha}}\, x] + C[2] \text{Sin}[\sqrt{\text{alpha}}\, x]]\}\}$

Problem 1

(b) Rearrange the equation like in the notes by expanding the derivative in the first term

```
eqb := beta * Exp[beta * x] * C'[x] + Exp[beta * x] * C''[x] == alpha * C[x]
```

```
eqb
```

$\text{beta}\, e^{\text{beta}\, x} C'[x] + e^{\text{beta}\, x} C''[x] == \text{alpha}\, C[x]$

```
DSolve[eqb, C, x]
```

$$\left\{\left\{C \to \text{Function}\left[\{x\}, -\frac{\sqrt{\text{alpha}}\sqrt{e^{-\text{beta}\, x}}\, \text{BesselI}\left[1, \frac{2\sqrt{\text{alpha}}\sqrt{e^{-\text{beta}\, x}}}{\text{beta}}\right] C[1]}{\text{beta}} + \frac{2\sqrt{\text{alpha}}\sqrt{e^{-\text{beta}\, x}}\, \text{BesselK}\left[1, \frac{2\sqrt{\text{alpha}}\sqrt{e^{-\text{beta}\, x}}}{\text{beta}}\right] C[2]}{\text{beta}}\right]\right\}\right\}$$

Problem 1 c

```
eqc := C''[x] - 10 * C'[x] == 2 * C[x]
```

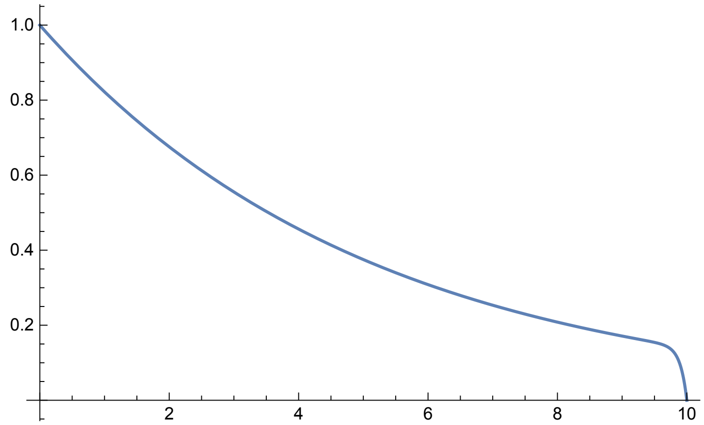
```
eqc
```

$-10\, C'[x] + C''[x] == 2\, C[x]$

```
sol = DSolve[{eqc, C[0] == 1, C[10] == 0}, C, x]
```

$$\left\{\left\{C \to \text{Function}\left[\{x\}, \frac{-e^{\left(5+3\sqrt{3}\right)x} + e^{60\sqrt{3}+\left(5-3\sqrt{3}\right)x}}{-1 + e^{60\sqrt{3}}}\right]\right\}\right\}$$

```
Plot[Evaluate[C[x] /. sol, {x, 0, 10}]]
```



Problem 1d

```
eqd := 3 * C'[x] + 3 * x * C''[x] == 12 * x
```

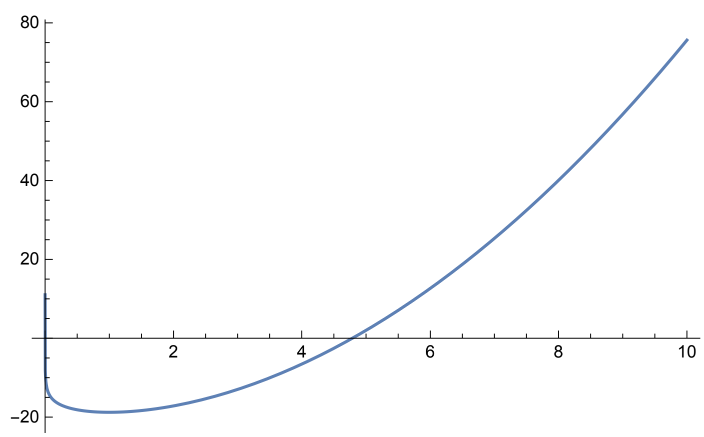
```
eqd
```

$3\,C'[x] + 3\,x\,C''[x] == 12\,x$

```
sol = DSolve[{eqd, C'[1] == 0, C[5] == 2}, C, x]
```

$\{\{C \to \text{Function}[\{x\}, -23 + x^2 + 2\,\text{Log}[5] - 2\,\text{Log}[x]]\}\}$

```
Plot[Evaluate[C[x] /. sol, {x, 0, 10}]]
```

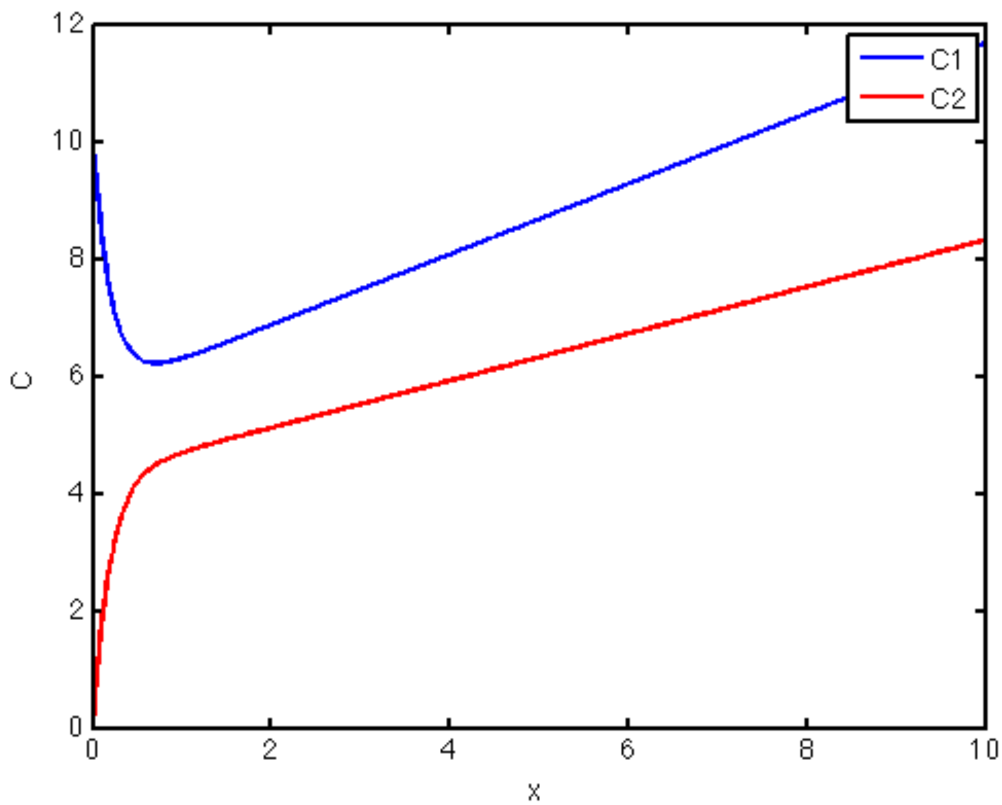


Below are the codes and plots that are generated by them. See below the figures for discussion.

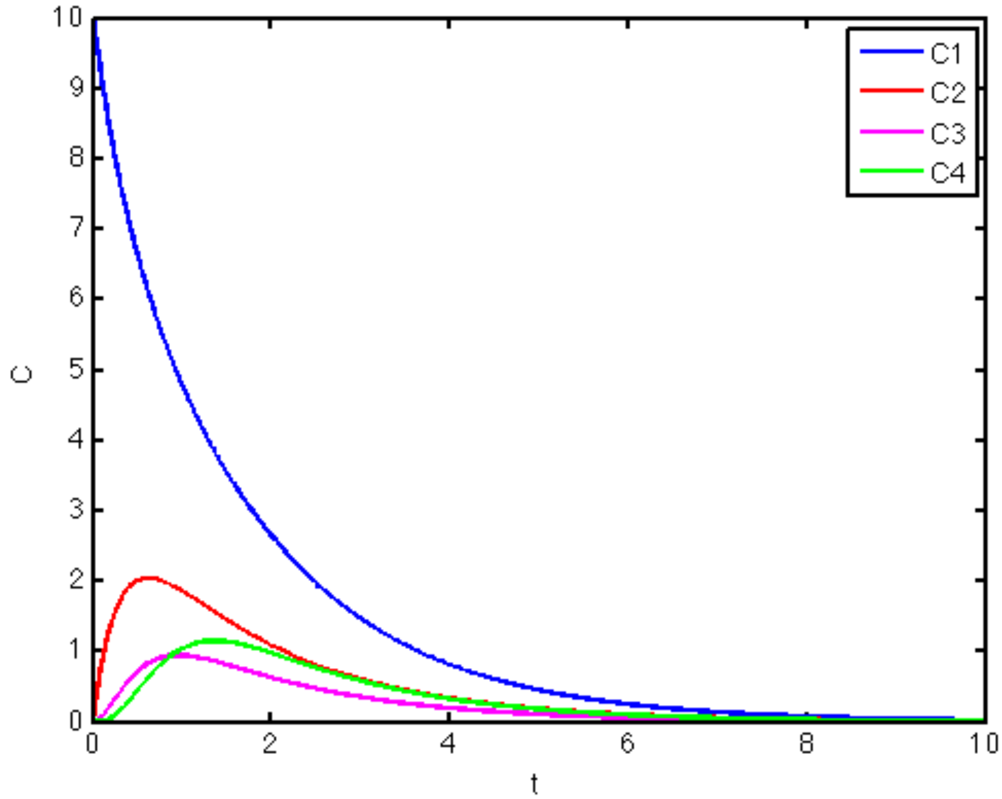
```
%HW1_Problem2

clear; clc; close all

%Part a
dx=0.01;
Nsteps=1000;

C1(1)=10;
C2(1)=0;

for kk=1:Nsteps
   C1(kk+1)=C1(kk)+dx*(-2*C1(kk)+3*C2(kk)-1);
   C2(kk+1)=C2(kk)+dx*(2*C1(kk)-3*C2(kk)+2);
end

x=0:dx:Nsteps*dx;
figure(1)
plot(x,C1)
hold on
plot(x,C2,'r')

%Part b
clear
dt=0.01;Nsteps=1000;

C1(1)=10;
C2(1)=0;
C3(1)=0;
C4(1)=0;

t=0:dt:Nsteps*dt;

for kk=1:Nsteps

   C1(kk+1)=C1(kk)+dt*(-C1(kk)+C2(kk));
   C2(kk+1)=C2(kk)+dt*(C1(kk)-3*C2(kk));
   C3(kk+1)=C3(kk)+dt*(2*C2(kk)-4*C3(kk));
   C4(kk+1)=C4(kk)+dt*(4*C3(kk)-3*C4(kk));

end

figure(2)
```

```
plot(t,C1)
hold on
plot(t,C2,'r')
hold on
plot(t,C3,'m')
hold on
plot(t,C4,'g')
```

Below are the plots



If you look at the equations, they are not in perfect balance. If you add them together the sources and sinks on the right hand side do not all cancel out. In fact you will have a positive source (2-1=1), which is why the concentrations keep increasing forever.



This system is well balanced and so will converge to a steady state unlike the last problem. Thus if you set all the time derivative terms to zero and solve the system of equations you will find that it all converges to concentration 0.