

Homework 4

$$\textcircled{1} D = 0.01 \text{ m}^2/\text{s}$$

$$H = 2.5 \text{ m}$$

$$V_s = 3.33 \times 10^6 \text{ r}^2$$

$$1 \mu\text{m} < r < 5 \text{ mm} \quad \Rightarrow \quad V_s = 3.33 \times 10^{-6} \text{ m/s} \quad (1 \mu\text{m})$$

$$= 83.25 \text{ m/s} \quad (5 \text{ mm})$$

$$C = C_0 e^{-\frac{V_s z}{D}}$$

$$\frac{C}{C_0} = e^{-\frac{3.33 \times 10^6 \text{ r}^2 z}{0.01}}$$

Lowest concentration at $z = H$

$$\Rightarrow \frac{C_{\min}}{C_0} = e^{-\frac{3.33 \times 10^6 \text{ r}^2 (2.5)}{0.01}}$$

Set $\frac{C_{\min}}{C_0} = 0.9$ as a minimum threshold

$$\Rightarrow 0.9 = e^{-\frac{3.33 \times 10^6 (2.5) \text{ r}^2}{0.01}}$$

$$\ln(0.9) = -8.325 \times 10^8 \text{ r}^2$$

$$r = 1.125 \times 10^{-5} \text{ m}$$

$$\approx 11 \mu\text{m}$$

② Power Law Fit

$$\dot{M} = 5.3 Q^{0.45}$$

$$= 5.3 (300 + 100 \sin(2\pi t))^{0.45}$$

$$\text{Mass} = \int_0^7 \dot{M} dt = 5.3 \int_0^7 (300 + 100 \sin(2\pi t))^{0.45} dt$$

$$= 480 \text{ tonnes}$$

③ $A = 750 \text{ m}^2$

$$V = 10^4 \text{ m}^3$$

$$K = 10^{-8} \text{ ms}^{-1}$$

C_{1a} ~ current air

C_{1w} ~ current water

C_{2a} ~ future air

C_{2w} ~ future water

$$V \frac{dC_w}{dt} = -K (C_w - \alpha C_a) A$$

Assume $C_a = C_{2a}$

$$\frac{dC_w}{dt} = -\frac{KA}{V} (C_w - \alpha C_{2a}) \quad C_w(t=0) = C_{1w}$$

Initially

$$C_{1w} = \alpha C_{1a}$$

$$C_{2w} = \alpha C_{2a}$$

$$\frac{C_{1w}}{C_{2w}} = \frac{C_{1a}}{C_{2a}} = 5$$

When is $C_w = \frac{1}{2} C_{1w}$?

$$C_w = \alpha C_{a2} + (C_{1w} - \alpha C_{a2}) e^{-\frac{KA}{V} t}$$

$$\frac{1}{2} C_{1w} = \alpha C_{a2} + (C_{1w} - \alpha C_{a2}) e^{-\frac{KA}{V} t}$$

$$C_{1w} = \alpha C_{1a} = 5 \alpha C_{a2}$$

$$\frac{5}{2} \alpha C_{a2} = \alpha C_{a2} + (5 \alpha C_{a2} - \alpha C_{a2}) e^{-\frac{KA}{V} t}$$

$$\frac{3/2}{4} = e^{-\frac{KA}{V} t}$$

$$t = \frac{V}{KA} \ln \left(\frac{8}{3} \right) = \frac{10^4}{(10^{-8})(750)} \ln \left(\frac{8}{3} \right) = 1.3 \times 10^9 \text{ s} \\ = 41.5 \text{ years}$$

$$K = 10^{-4} \text{ m/s} \quad = \frac{10^4}{10^{-4}(750)} \ln \left(\frac{8}{3} \right) = 1.3 \times 10^5 \text{ s}$$

$$= 1.5 \text{ days}$$