Sample Problem

$M = 10\ kg$

$q = 2\ m/day$

$n_e = 0.3$

$$u = \frac{2}{0.3} = \frac{20}{3}\ m/day = 6.67\ m/day$$

$\alpha_L = 0.01 m \Rightarrow D_x = 0.0667\ m^2/day$ (neglecting diffusion)

$\alpha_T = 0.001 m \Rightarrow D_y = 0.00667\ m^2/day$ (neglecting diffusion)

↳ just about o.k.!

$$\therefore C(x,y,t) = \frac{M}{4\pi (D_x D_y)^{1/2} t} e^{-\frac{(x-x'-ut)^2}{4D_x t}} e^{-\frac{(y-y')^2}{4D_y t}}$$

$$= \frac{10}{4\pi(0.02)t} e^{-\frac{(x-6.67t)^2}{4(0.0667)t}} e^{-\frac{(y)^2}{4(0.00667)t}}$$

$$= \frac{39.8}{t} e^{-\frac{(x-6.67t)^2}{0.27t}} e^{-\frac{y^2}{0.027t}}$$

Set $x = 200$ and $y = -20, -10, 0, 10, 20$

and plot

No the area under the curves is not constant, because only part of the mass passes any point

Sample Problem

$q = 2\ m/day$

$n = 0.25$

$\Bigg\}\ u = 8\ m/day$

$\alpha_x = 0.01$

$\alpha_y = 0.001$

Steady State $\Rightarrow$ $C = \dfrac{C_0}{2} e^{\frac{x}{2\alpha_x}\left(1 - \sqrt{\frac{u + 4\gamma\alpha_x}{u}}\right)}$

$$\left[\operatorname{erf}\left(\frac{y + L/2}{2\sqrt{\alpha_y x}}\right) - \operatorname{erf}\left(\frac{y - L/2}{2\sqrt{\alpha_y x}}\right)\right]$$

Set $y = 0$, $\gamma = 0$, $L = 50$, $\alpha_x = 0.01$, $\alpha_y = 0.001$

$$\Rightarrow C = \frac{C_0}{2}\left[\operatorname{erf}\left(\frac{L}{4\sqrt{\alpha_y x}}\right) - \operatorname{erf}\left(\frac{-L}{4\sqrt{\alpha_y x}}\right)\right]$$

$$= C_0\left[\operatorname{erf}\left(\frac{L}{4\sqrt{\alpha_y x}}\right)\right]$$ (because erf is antisymmetric)

Where is $C = \dfrac{C_0}{10^4}$

$$\Rightarrow 10^{-4} = \operatorname{erf}\left(\frac{50}{4\sqrt{10^{-3} x}}\right)$$

$$\frac{50}{4\sqrt{10^{-3}x}} = \text{inv erf}\,(10^{-4})$$

$$\Rightarrow \quad x = \frac{50^2}{4^2\left(\text{inverf}(10^{-4})\right)^2}\,\frac{1}{10^{-3}} = 2\times10^{13}\ \text{m}$$

↳ awsome

↓ Matlab function erfinv

(ii) $$C = \frac{C_o}{2}\, e^{\frac{x}{2\alpha_x}\left(1-\sqrt{\frac{u+4\gamma\alpha_x}{u}}\right)}\left[\operatorname{erf}\left(\frac{y+L/2}{2\sqrt{\alpha_y x}}\right)-\operatorname{erf}\left(\frac{y-\frac{L}{2}}{2\sqrt{\alpha_y x}}\right)\right]$$

Set $y=0$, $\gamma=?$, $L=50$, $\alpha_x=10^{-2}$, $\alpha_y=10^{-3}$; $x=200$

$$C = \frac{C_o}{10^4}$$

$$\Rightarrow \quad C_o\,10^{-4} = \frac{C_o}{2}\, e^{\frac{200}{2\times10^{-2}}\left(1-\sqrt{1+\frac{0.04\gamma}{8}}\right)}\left[2\operatorname{erf}\left(\frac{50}{4(0.2)^{1/2}}\right)\right]$$

$$C_o\,10^{-4} = C_o\, e^{10^4\left(1-(1+5\times10^{-3}\gamma)^{1/2}\right)}$$

$$\ln(10^{-4}) = 10^4\left(1-\sqrt{1+5\times10^{-3}\gamma}\right)$$

$$1+5\times10^{-3}\gamma = \left(1-\frac{\ln(10^{-4})}{10^4}\right)^2$$

$$\gamma = 0.3686\ \text{day}^{-1}$$