Probabilistic risk analysis of building contamination

Abstract We present a general framework for probabilistic risk assessment (PRA) of building contamination. PRA provides a powerful tool for the rigorous quantification of risk in contamination of building spaces. A typical PRA starts by identifying relevant components of a system (e.g. ventilation system components, potential sources of contaminants, remediation methods) and proceeds by using available information and statistical inference to estimate the probabilities of their failure. These probabilities are then combined by means of fault-tree analyses to yield probabilistic estimates of the risk of system failure (e.g. building contamination). A sensitivity study of PRAs can identify features and potential problems that need to be addressed with the most urgency. Often PRAs are amenable to approximations, which can significantly simplify the approach. All these features of PRA are presented in this paper via a simple illustrative example, which can be built upon in further studies.

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Practical Implications
The tool presented here can be used to design and maintain adequate ventilation systems to minimize exposure of occupants to contaminants.

Introduction

Accurate and verifiable predictions of air flow and contaminant transport in buildings are notoriously tedious because of complexities introduced by turbulent flows and spatial geometries within typical buildings. The lack of sufficient site characterization, computational resources, and occasionally, the inadequacy of conceptualizations and mathematical models of relevant physical phenomena complicate modeling efforts further. These problems introduce a fundamental lack of certainty about the flow in a room and cast doubts on the feasibility of obtaining a single deterministic prediction of a building's response to contamination.

Most dynamic models of the physical phenomena arising in the ventilation disciplines has been carried out within a deterministic framework. That is, given sufficient information at one instant in time, the deterministic model determines 'exactly' the entire future behavior of the system. In reality, complex room layout and air distribution systems can introduce a significant amount of uncertainty. Even in a fully

deterministic system, different ventilation schemes can give rise to significant concentration gradients – e.g. Hunt and Kaye (2006) and Bolster and Linden (2007) showed that under conditions of idealized displacement, ventilated space concentrations can vary over several orders of magnitude. Local conditions such as occupant behavior can also significantly affect local concentrations. Ozkaynak et al. (1982) found that pollutant levels in a kitchen with the oven on depended heavily on the sampling location. Rodes et al. (1991) discovered that personal sampling almost always reveals a higher exposure to contaminants that would be predicted from indoor air monitoring that assumes perfect mixing. Lambert et al. (1993) showed that the levels of suspended particles in a non-smoking section of a restaurant were 40–65% less than those in the smoking section.

Some of these critical phenomena can be analyzed by means of computational fluid dynamics (CFD). However, detailed full-scale simulations can be prohibitively expensive, particularly for large buildings with multiply connected spaces. Moreover, accurate CFD simulations of many ventilation flows can be elusive, because

of uncertain boundary conditions (Cook et al., 2003) and/or the difficulty of selecting an appropriate mathematical description from a wide choice of turbulence models available (Ji et al., 2007).

Probabilistic models provide an alternative to deterministic descriptions of ventilation phenomena by assuming that regardless of how much is known about a system at a given instant, it is impossible to determine with absolute certainty the future behavior of the system. It is common to distinguish two types of uncertainty: epistemic and aleatory (Tartakovsky, 2007 and references therein). The former designates uncertainty introduced by incomplete knowledge about system parameters (the so-called parametric uncertainty) and/or driving forces, i.e. initial and boundary conditions, and conceptual models (the so-called model or structural uncertainty); it can be reduced or completely eliminated by knowledge-enhancing observations (data). The latter refers to uncertainty associated with the random nature of a phenomenon, e.g. turbulent dispersion of a contaminant; it is inherent to the system and cannot be eliminated by observation. One probabilistic approach is to model systems with stochastic, rather than deterministic, governing equations (e.g. Flynn, 2004; Haghighat et al., 1988; Siurna et al., 1989). While stochastic modeling can be appealing from a scientific perspective, it involves some 'heavy mathematical weightlifting' that can act as a deterrent to its practical implementation. An essential communication of important results is often inaccessible to the non-expert. Moreover, it can often be difficult to efficiently apply stochastic models outside of the realm of idealized situations because of the complexity of the practical application.

We suggest an alternative method, namely probabilistic risk analysis (PRA), which can act as a translator between scientists, engineers, investors, politicians and decision-makers. A PRA starts from an uncertain perspective, which, in principle, allows it to account for both epistemic and aleatory uncertainties. PRA can be used in parallel or instead of CFD, and can be integrated into zonal computer models that treat buildings as a connected network of spaces that interact with one another (e.g. the US Department of Energy code 'EnergyPlus'). Once properly formalized, PRA can be effectively implemented and it is used in a range of other engineering disciplines.

In the aerospace sector a systematic approach towards risk assessment has been common practice since the disastrous Apollo test in 1967. Currently, NASA and the European Space Agency (ESA) adopt PRA methods to understand, quantify, and minimize the risks involved in space travel (e.g. Colglazier and Weatherwas, 1986; ESA, 1997; Fragola, 1995; Garrick, 1989). The nuclear sector, where fears and consequences of failure are large, adopted these methods after the Three Mile Island accident. The US Nuclear Regulatory Comission (NRC) has since introduced and developed many powerful techniques to identify and eliminate the risks involved (e.g. Garrick, 1984; NRC, 1983; Vesely et al., 1981).

In sectors, somewhat closer to building contamination and ventilation, many risk-based approaches have been adopted in modeling techniques. For example, these methods are commonly used when studying fire safety in buildings (Chua et al., 2007; Frantzich, 1998) or assessing a building's resilience to seismic activity (Ellingwood, 2001). In the field of contamination in buildings there is a clear interest in assessing the risks of contamination. Recent studies which focus on assessing risks associated with contaminants include those of Gravesen et al. (1999), Jaakkola and Miettinen (1995), Milton et al. (2000), and Mizoue et al. (2001), among many others. However, most of these investigations are either case studies or statistical analyses of very specific scenarios.

In this paper, we introduce a general framework for PRA that ties statistical and theoretical studies in a way that is more accessible and useful to practitioners and decision-makers than that routinely used by scientists, including the stochastic differential equation methods discussed above. While this framework is applicable to a variety of contamination problems, we introduce the basic ideas and concepts by considering a very simple example described in the following section.

Probabilistic risk analysis

The concentration of a contaminant in a space, C, is fundamental in determining the environmental health risks. For example, if the contaminant in question is carcinogenic, its impact on human health can be quantified by the Excess Lifetime Cancer Risk (ELCR) factor (EPA, 1992)

$$\text{ELCR} = \sum_{i=1}^{n} \text{ISF} \times \text{CDI}_i, \tag{1}$$

where

$$\text{CDI}_i = \frac{\bar{C}_i \times \text{IR}}{\text{BW}}, \quad \bar{C} = \frac{1}{24}\int_0^{24} C(t)\mathrm{d}t,$$
$$\text{ISF} = \frac{\text{IUR} \times \text{BW}}{\text{IR}} \tag{2}$$

where IR is the human inhalation rate, $\bar{C}$ the average exposure concentration over 1 day, BW is average body weight and IUR is the inhalation unit risk (or inhalation slope factor). While any or all parameters of the expressions given above can be uncertain, it is common to take IR $= 20$ m^3/day and BW $= 70$ kg and check for IUR in the appropriate tables. According to the EPA, the level of a carcinogen (e.g. asbestos) in the air is considered safe (i.e. health risks are

assumed acceptable) if the ELCR $< 10^{-6}$. If the ELRC $> 10^{-3}$, then the situation is deemed serious and is a high priority for attention.

Additionally, there are maximum levels of certain contaminants that people should not be exposed to even over short durations. A key goal of PRA is to determine the probability that the carcinogen's concentration C in a room exceeds, at time $t = T$, the EPA-mandated value of $C^* = \alpha^{-1}$ELCR.

Most present-day analyses of building contamination lack PRA and uncertainty quantification of the kind routinely required in other engineering and environmental disciplines (Covello and Mumpower, 1985). Specifically, a comprehensive risk analysis should provide answers to the following three questions: 'What can happen?' 'How likely is it to happen?' 'Given that it occurs, what are the consequences?' (Bedford and Cooke, 2003). Several recent studies (Haghighat et al., 1988; Hammitt et al., 1999; Spicer, 2000) addressed some of these questions, but not all.

A typical building space has a complex geometry, variable occupational density, and a number of distributed contamination sources. We consider contaminants released via internal or external sources that migrate within the space in question (Figure 1). We say that the ventilation system 'fails' at time $t = T$, if the contaminant concentration exceeds some mandated level of C^*. Our objective is to assess both the likelihood of the system failure and the efficiency of alternative remediation strategies.

Following Bedford and Cooke (2003), we start by constructing a 'fault tree' (Figure 2), which relates the occurrence of the system failure with the failures of its constitutive parts (basic events), i.e. the occurrence of a contaminant release, the failure of a filter, the failure of natural attenuation, and the failure of a remediation effort. The term 'natural attenuation' is used here to describe the usual transport mechanisms (mainly advection) that enable the contaminant to leave the room. The Boolean operators 'AND' and 'OR' indicate a collection of b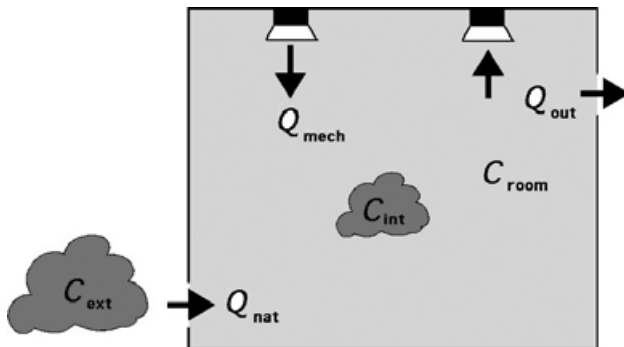asic events that would lead to room contamination. The example presented here is not meant to be an exhaustive description of all possible contamination events, but merely a simple illustrative example.

The second step is to identify 'minimal cut sets' of the system, i.e. the smallest collections of events that lead to building contamination. The fault tree in Figure 2 reveals three such minimal cuts: {ICR,VF}, {ICR,FS} and {ECR,FF}. (See Table 1 for a definition of the abbreviations.)

The third step is to represent the fault tree in Figure 2 by a Boolean expression. Recalling that the Boolean operators AND and OR applied to two events X and Y can be written as X AND $Y \equiv X \cdot Y \equiv X \cap Y$ and X OR $Y \equiv X + Y \equiv X \cup Y$, the Boolean expression corresponding to the fault tree in Figure 2 is

$$
\begin{aligned}
\text{SF} &= \text{ICR} \cdot \text{AF} + \text{ECR} \cdot \text{FF} \\
&= \text{ICR} \cdot (\text{VF} + \text{FS}) + \text{ECR} \cdot \text{FF} \quad (3) \\
&= \text{ICR} \cdot \text{VF} + \text{ICR} \cdot \text{FS} + \text{ECR} \cdot \text{FF}.
\end{aligned}
$$

The latter expression is known as a 'cut set representation' of the fault tree in Figure 2.

The final step is to use Equation 3 to compute $P[\text{SF}]$, the probability of building contamination at time $t = T$,

$$
\begin{aligned}
P[\text{SF}] =\ & P[(\text{ICR} \cap \text{VF}) \cup (\text{ICR} \cap \text{FS}) \cup (\text{ECR} \cap \text{FF})] \\
=\ & P[\text{ICR} \cap \text{VF}] + P[\text{ICR} \cap \text{FS}] + P[\text{ECR} \cap \text{FF}] \\
& - P[\text{ICR} \cap \text{VF} \cap \text{FS}] \\
& - P[\text{ICR} \cap \text{ECR} \cap \text{VF} \cap \text{FF}] \\
& - P[\text{ICR} \cap \text{ECR} \cap \text{FS} \cap \text{FF}] \\
& + P[\text{ICR} \cap \text{ECR} \cap \text{VF} \cap \text{FS} \cap \text{FF}]. \quad (4)
\end{aligned}
$$

Assuming that all the events at the bottom of the fault tree are independent, the probability of building contamination can be simplified to yield

$$
\begin{aligned}
P[\text{SF}] =\ & P[\text{ICR}](P[\text{VF}] + P[\text{FS}] - P[\text{VF}]P[\text{FS}]) \\
& + P[\text{ECR}]P[\text{FF}] + P[\text{ICR}]P[\text{ECR}] \\
& (P[\text{VF}]P[\text{FF}]P[\text{FS}] - P[\text{VF}]P[\text{FF}] \\
& - P[\text{FS}]P[\text{FF}]). \quad (5)
\end{aligned}
$$

Definition of probabilities

The procedure described above is used extensively in PRA of complex systems, such as nuclear power plants and space shuttles (Bedford and Cooke, 2003). In order to use it in building ventilation we must carefully consider how to evaluate the probabilities entering Equation 5.

Modern PRAs utilize *subjective* probability, which is defined as the 'degree of belief, of one individual' (Bedford and Cooke, 2003). Subjective probabilities in
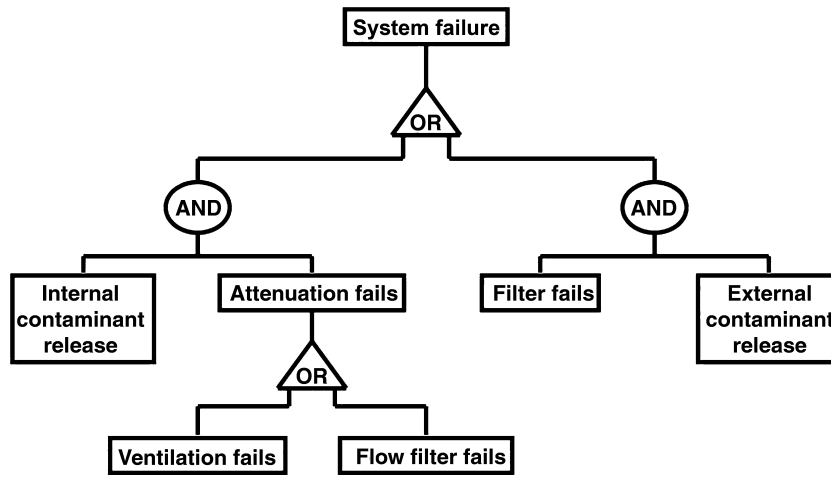

Fig. 1 Contamination scenarios for a hybrid ventilated space

Fig. 2 Fault tree for a possible building contamination

Table 1 Glossary of event abbreviations

Event	Abbreviation
System failure	SF
Internal contaminant release	ICR
External contaminant release	ECR
Attenuation fails	AF
Filter fails	FF
Ventilation fails	VF
Flow filter fails	FS

building ventilation must be used to deal with both parametric and structural (model) uncertainties, as many fundamental issues in ventilation flows and transport are yet to be resolved. This is illustrated, for example, by the ongoing debates on whether contaminant transport can be accurately represented by the perfect mixing assumption and whether flows can be adequately described with simple turbulence models. Our subjective belief is that the answer to the first question is negative and that to the second one is positive, but other opinions, and mathematical models based on these opinions, should be incorporated into a rigorous PRA to avoid a systematic predictive bias.

In other words, the ability to incorporate expert knowledge into quantitative ventilation modeling becomes paramount. Several formal approaches to dealing with diverging expert opinions, originally developed in economics (Otway and Winterfeldt, 1992), can be adapted in ventilation, just as it has been done in other fields (NRC, 1997). (Their practical implementation is a separate topic that is beyond the scope of this paper.)

Probabilistic risk assessments used in most engineering applications rely on reliability databases and manufacturing specifications to estimate the probabilities of basic events. Modern reliability data banks are used first by design engineers to insure optimum initial performance, then by maintenance engineers to design a proper maintenance schedule so as to maintain optimum performance throughout the system's lifetime, and finally by risk analysts to quantify the reliability of the system. Examples of reliability data banks for engineering components include Failure Rate Data in Perspective (FARADIP; Smith, 2001), Offshore Reliability Data Handbook (OREDA; OREDA, 2002), and Nonelectronic Parts Reliability Data (NPRD) (Denson, 1995). Many of these components make up HVAC plants, and the databases can be used to compile reliability data for specific plants. Reliability data more specific to HVAC is more limited, but does exist (e.g. Hale and Arno, 2001; Hollis et al., 2000). Additional datasets should be initiated and updated.

In ventilation, reliability databases can be supplemented with analytical models, such as a well-mixed model for simple ventilation systems and geometries Nazaroff and Cass (1991) or more complex models for more complex systems that allow for the presence of concentration gradients (Bolster and Linden, 2007; Hunt and Kaye, 2006). Another alternative is to incorporate stochastic techniques (Haghighat et al., 1988) to compute probabilities quantifying structural and parametric uncertainties associated with the ventilation processes. The uncertainties typically arise from an imperfect knowledge of the flow into and within a space, which is essential in estimating the transport of contaminants within a building. The computed probabilities can then be used to compute the probabilities of the basic events in Equation 4.

The practical implementation of statistical inference relies on observations of a random phenomenon to choose the probability distribution that best describes it. A Bayesian approach provides a natural framework to achieve this goal. First, expert knowledge is used to select a *prior*, i.e. the form of probability distribution

function (PDF) that, in the expert's opinion, is most appropriate to a given situation. The Bayesian philosophy states that the *prior* is to be selected without first looking at the data. Secondly, Bayes' theorem is used to construct the *posterior* distribution, the updated PDF accounting for the available data. This Bayesian viewpoint is popular in the risk analysis community, because data from many different sources can be combined to evaluate the *posterior*. Additionally, the Bayesian approach allows one to refine the *posterior* distributions as new datasets are acquired.

The sample problem described by the fault tree in Figure 2 contains several probabilities that need to be determined. For example, the probability of failure of the filters depends on their life time, i.e. the longer the filter operates, the more likely it is to fail. Similarly, the ventilation has a lifetime component that depends upon both the mechanical system and a variation of penetrative flow caused by varying environmental conditions. Finally, one has to determine the probability of a contaminant release.

Filter probabilities

Probability of failure of many manufactured components, including filters, can often be described by the exponential cumulative distribution function,

$$F_X(t) = P[X \le t] = 1 - e^{-\lambda t}, \tag{6}$$

which is the probability that a component with lifetime X has failed before time t. The reliability function, $R_X(t) = 1 - F_X(t)$, represents the probability that a component with lifetime X is still working at time t. (Note that exponential distributions should only be applied to systems with non-negative values of X.) The parameter $1/\lambda$ represents the expected or average life of a component. It can be estimated from either manufacturers' specification or reliability tables for the specific application or maintenance data.

We assume exponential distributions for both filters, which means that

$$F_{FF} = 1 - e^{\lambda_{ff} t}, \quad F_{FS} = 1 - e^{\lambda_{fs} t}. \tag{7}$$

The values of λ_{ff} and λ_{fs} should be estimated using Bayesian inference with manufacturers' guidelines and any available data. They will also depend on the specific application of the filter. For example, certain manufacturers' data state that the average lifetime of internal filters in a restaurant kitchen will vary from 30 days in a wood-fire kitchen (i.e. $\lambda = \frac{1}{30}$ per day), to 60 days in a greasy fast food kitchen (i.e. $\lambda = \frac{1}{60}$ per day), to 1 year for kitchens where items are predominantly boiled (i.e. $\lambda = 1/\text{year}$). Therefore, it is important not only to consider the filters being used, but also their specific application. Another complication,

which can be addressed within our proposed framework, is the increased probability of filters' failure because of mechanical stresses caused by their testing.

It is worthwhile noting that the exponential distributions similar to Equation 7 are popular in reliability and risk assessment, because they have only one fitting parameter λ, which makes them easier to use. More complex distributions, such as the Weibul distribution, can be used instead (Bedford and Cooke, 2003). Among the effects contributing to a filter's failure, but not accounted for in Equation 7, is the deterioration of its performance with time. If this effect is important, one can define the failure either as a complete failure or a situation in which a filter functions below a certain performance efficiency.

Ventilation probabilities

When considering ventilation, it is important to account for both mechanical and natural (penetrative or open windows) sources. The flow rate from the mechanical system is unlikely to fluctuate much about the programmed value. However, because it is a complex mechanical system there is always a probability of failure associated with it. For simplicity, we again assume an exponential distribution

$$F_{Q_m} = 1 - e^{-\lambda_m t}. \tag{8}$$

The expected lifetime λ_m must be chosen from manufacturers' guidelines and will also depend on the specific application and type of HVAC system. For example, according to the Consortium for Energy Efficiency (CEE), a cheap single-room HVAC system can have an average lifetime as low as 3 years ($\lambda_m = \frac{1}{3}$ per year). However, most typical single-room HVAC systems have an average lifetime of about 10 years ($\lambda_m = \frac{1}{10}$ per year) and a larger central system can have an average lifetime of 18 years (Rosenquist et al., 2001), giving $\lambda_m = \frac{1}{18}$ per year.

On the other hand, the natural ventilation component can vary greatly depending on external and internal conditions. It is likely to have a mean value around which it fluctuates and so we assume a normal (Gaussian) cumulative distribution function

$$F_X(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{t} \exp\left[\frac{-(u-\mu)^2}{2\sigma^2}\right] du. \tag{9}$$

The distribution parameters μ and σ, representing the mean and the standard deviation, respectively, are to be determined from the available data. Both positive and negative values of X are possible, as the 'bell-shaped' curve extends to $\pm\infty$.

It is important to realize the practical limitations imposed by any choice of a probability model, including

the Gaussian cumulative distribution function (9). For example, natural ventilation can depend heavily on external conditions (e.g. temperature and wind) that can vary substantially over time. The Gaussian model (9) does not account for these effects; this choice is based on both previous practice (Haghighat et al., 1988) and its relative simplicity. A different distribution, based on another expert opinion and local weather conditions, may be more appropriate depending on the circumstances. Alternatively, to account for seasonal variations, multiple distributions, again based on expert opinion, with different mean values and variations depending on the time of year could be used.

With this caveat, we define the probability of failure of the ventilation system based on a minimum required flow rate, i.e. $Q < Q_{min} \Rightarrow$ FAILURE. This minimum flow rate must be specified and should at least be greater than the minimum standards or recommendations specified by organizations such as ASHRAE (Standard 62-2001), CIBSE (http://www.cibse.org), EPA (http://www.epa.gov) and NIST (http://www.nist.gov). Results from previous analytical and experimental studies (Bolster and Linden, 2007; Hunt and Kaye, 2006; Nazaroff and Cass, 1991) can be used to estimate Q_{min} more accurately.

Let us assume that the mechanical system provides a constant flow rate (any perturbations are considered small relative to the mean). The flow Q_m can take on two values: either $\bar{Q}_m$ when it is functioning or 0 after the system fails. Additionally, there is the natural flow rate Q_{nat} as illustrated in Figure 1. The probability of failure for this system is given by

$$
\begin{aligned}
P[Q_{nat} < Q_{min} - Q_m] &= P[Q_{nat} < Q_{min} - \bar{Q}_m] \\
&\times P[Q_m \neq 0] \\
&+ P[Q_{nat} < Q_{min}] \\
&\times P[Q_m = 0],
\end{aligned}
\tag{10}
$$

where

$$
P[Q_m = 0] = 1 - P[Q_m \neq 0].
\tag{11}
$$

The probability of the flow rate being too small is

$$
\begin{aligned}
P[VF] &= P[Q_{nat} < Q_{min} - \bar{Q}_m]P[Q_m \neq 0] \\
&+ P[Q_{nat} < Q_{min}]P[Q_m = 0].
\end{aligned}
\tag{12}
$$

Using the exponential distribution (8) to model the mechanical ventilation system and the normal distribution (9) to model the natural flow rate, we obtain

$$
P[VF] = F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t}),
\tag{13}
$$

where $F_1 = F_{nat}(Q_{min} - Q_m)$, $F_2 = F_{nat}(Q_{min})$, and

$$
\begin{aligned}
F_{nat}(A) &\equiv P[Q_{nat} < A] \\
&= \frac{1}{\sigma_{nat}\sqrt{2\pi}} \int_{-\infty}^{A} \exp\left[\frac{(u - \bar{Q}_{nat})^2}{2\sigma_{nat}^2}\right] du.
\end{aligned}
\tag{14}
$$

Contaminant release probability

The probability of a contaminant release depends on the type of the contaminant and the contamination scenario. One must consider both the internal (ICR) and external (ECR) release situations. Knowledge of the nature of the potential contaminant sources is critical in estimating the probability of occurrence of the contamination. One can define this probability as

$$
P[CR] = \frac{1}{T} \int_0^T H[C_s(t) - C_{min}] dt,
\tag{15}
$$

where $H(x)$ is the Heaviside step function defined as $H(x > 0) = 1$ and $H(x < 0) = 0$; T is the sampling period, which should be sufficiently long to cover all typical contamination situations; and $C_s(t)$ is the strength of the contaminant sources. The minimum source strength C_{min} is included into Equation 15 to account for the fact that it is 'dose that makes the poison' (Ottoboni, 1997), i.e. that the contamination with levels below C_{min} is inconsequential. The value of C_{min} can be specified from standards or from other concepts, including the ELCR described in Equation 1. Zero tolerance for a particular contaminant corresponds to setting $C_{min} = 0$.

It is important to note that with the current approach the probability of contamination is a constant number. One can also account for a time dependence of this probability based on time of day, week or year, as these can be important factors in determining the likelihood of a contamination release.

As an example of contamination probabilities, consider the following four types of sources of contaminants:

Continuous sources: If a contaminant is released continuously, then the probability of contamination is, of course, $P[C] = 1$.

Partially continuous sources: If the contaminant (e.g. carbon dioxide) is released into an occupied office building only during business hours (e.g. by occupants exhaling CO_2 during 8 h in the office), the probability of contamination is close to $P[C] = 0.33$. (On the other hand, if one is concerned with keeping the space contaminant free during hours of occupation, then CO_2 will fall into the category of continuous sources.)

Intermittent sources: This category includes other contaminants, such as exhaust emissions from outside traffic or contaminants associated with intermittent

activities. For these, the probability of contamination must be estimated from experimental data and knowledge of outside and indoor activities and conditions.

Rare sources: If the contaminant in question is anthrax from a terrorist attack in a standard office building, a nuclear spill at a nearby power plant, or any other event that does not happen on a regular basis, then the probability of contamination is very small $P[C] = \varepsilon$, where $\varepsilon \sim 0$. Expert opinion typically informs this probability.

This line of reasoning allows one to estimate the probabilities of internal and external contamination (15),

$$P[\text{ICR}] = C_{\text{icr}}, \quad P[\text{ECR}] = C_{\text{ecr}}. \tag{16}$$

Inserting these probabilities into Equation 5, we obtain the probability of system failure (i.e. building contamination)

$$
\begin{aligned}
P[\text{SF}] = &\; C_{\text{icr}}\{F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t}) + 1 - e^{-\lambda_{\text{fs}} t} \\
&- [F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t})](1 - e^{-\lambda_{\text{fs}} t})\} \\
&+ C_{\text{ecr}}(1 - e^{-\lambda_{\text{ff}} t}) + C_{\text{icr}} C_{\text{ecr}}\{[F_1 e^{-\lambda_m t} \\
&+ F_2(1 - e^{-\lambda_m t})](1 - e^{-\lambda_{\text{fs}} t})(1 - e^{-\lambda_{\text{ff}} t}) \\
&- [F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t})](1 - e^{-\lambda_{\text{ff}} t}) \\
&- (1 - e^{-\lambda_{\text{fs}} t})(1 - e^{-\lambda_{\text{ff}} t})\}. \tag{17}
\end{aligned}
$$

Parameter sensitivity study

Now that we have found the probability of failure of the system, it is important to understand how each parameter influences the likelihood of failure. The first thing to note is that the probability of system failure is a function of time that tends to some steady-state value at large times. This is because eventually all components (filters and the mechanical ventilation) will fail and so the system ultimately fails when a contaminant release occurs. The probabilities of contaminant release, C_{icr} and C_{ecr}, do not significantly influence the rate at which the probability of system failure occurs. Their primary influence is on the magnitude and the ultimate steady-state value, $P[\text{SF}](t \to \infty) = C_{\text{icr}} + C_{\text{ecr}} - C_{\text{icr}} C_{\text{ecr}}$. Similarly, the values of F_1 and F_2 do not affect the rate at which the probability changes. These are solely determined by the parameters λ, i.e. by the mean lifetime of the components with the exponential distributions. This is because only these components have a time-dependent probability. The natural flow rate, the probability of contaminant release, and the average mechanical flow rate are assumed to be constant in time.

Figure 3 illustrates how the probability of system failure changes for different values of system parameters. The values of the probabilities of internal (C_{icr}) and external (C_{ecr}) contamination influence the magnitude and the final steady-state values of the probability of contamination. The probability that the mechanical system provides a sufficient flow rate, F_1,
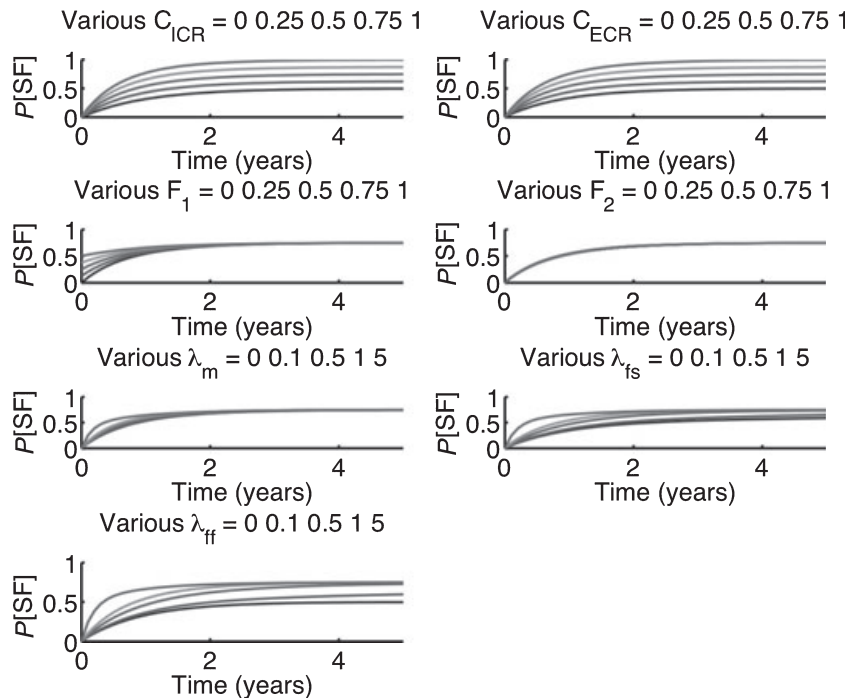

Fig. 3 Probability of building contamination, $P[\text{SF}]$, for a range of system parameters. In each figure, one of the parameters is varied, while the rest are kept at their reference values of $C_{\text{ICR}} = 0.5$, $C_{\text{ECR}} = 0.5$, $F_1 = 0.01$, $F_2 = 0.9$, $\lambda_m = 0.1$, $\lambda_{\text{fs}} = 1$ and $\lambda_{\text{ff}} = 1$

affects the initial value of the probability of contamination. This is because the system will eventually (i.e. at steady state) fail. However, for a well-designed system, F_1 will be small and therefore the initial probability of failure will be small too. However, if the initial system design is inadequate, the initial failure rate will be high. The system is fairly insensitive to F_2, the probability that natural ventilation alone will provide ample ventilation for contaminant removal.

As expected, the exponential constants for the failure of the mechanical system and filters have the most influence on the system behavior. The influence of λ_m seems relatively small. This is because regardless of how good the ventilation system is, the filters will typically fail first, thus causing the system to fail regardless of the fact that the mechanical system is still working. This is evident in the plots for λ_{ff} and λ_{fs}, which show that at any given time the filters with a longer average lifetime result in significantly lower probabilities of failure.

The sensitivity of the probability of failure to each of the system parameters can be elucidated by analyzing the derivative of $P[SF]$ in Equation 17 with respect to each of the parameters,

$$\frac{dP[SF]}{d\lambda_m} = \frac{dP[VF]}{d\lambda_m}P[ICR]\{1 - P[FS] + P[ECR](P[FF]P[FS] - P[FF])\} \quad (18)$$

$$\frac{dP[SF]}{d\lambda_{ff}} = \frac{dP[FF]}{d\lambda_{ff}}P[ECR]\{1 + P[ICR](P[VF]P[FS] - P[VF] - P[FS])\} \quad (19)$$

$$\frac{dP[SF]}{d\lambda_{fs}} = \frac{dP[FS]}{d\lambda_{fs}}P[ICR]\{1 - P[VF] + P[ECR](P[VF]P[FF] - P[FF])\} \quad (20)$$

$$\frac{dP[SF]}{dF_1} = \frac{dP[VF]}{dF_1}P[ICR]\{1 - P[FS] + P[ECR](P[FF]P[FS] - P[FF])\} \quad (21)$$

$$\frac{dP[SF]}{dF_2} = \frac{dP[VF]}{dF_2}P[ICR]\{1 - P[FS] + P[ECR](P[FF]P[FS] - P[FF])\}. \quad (22)$$

Figure 4, which displays these derivatives, makes it clear that the probability of contamination is most sensitive to F_1, especially at early times. Recall that this parameter determines the initial condition on the probability of failure; the smaller F_1, the better the initial design. The probability of contamination is least sensitive to F_2, the probability that natural ventilation
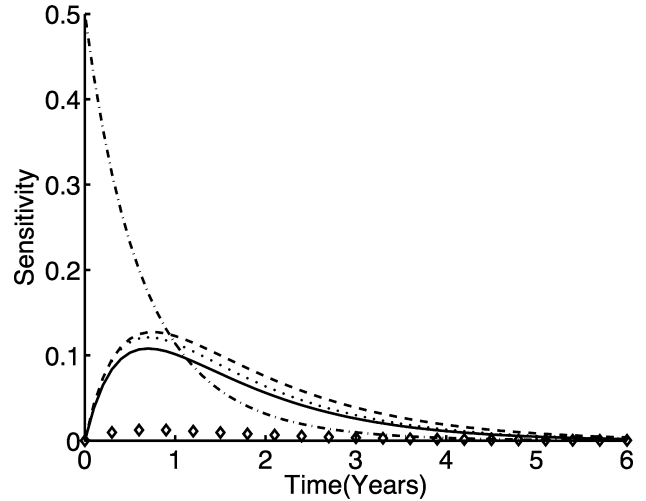

Fig. 4 Sensitivity of the probability of contamination to each of the system parameters. F_1 (−·), F_2 (◇), λ_m (−), λ_{ff} (- -) and λ_{fs} (·)

alone will provide ample ventilation for contaminant removal. The sensitivity of the probability of failure to the three exponential coefficients persistent most, reaching its maximum at about 1 year after the start of operations.

This sensitivity analysis suggests the following approach to minimizing contaminant exposure. The initial design must insure adequate contaminant removal. Otherwise, the system has an appreciable probability of failing right from the beginning. After this, it is most important to use high-quality filters with long lifetimes. Finally, proper maintenance of the ventilation system is required to avoid exposure. However, because of the robustness and typical average lifetimes of a ventilation system, it is not necessary to maintain it as often as it is to replace filters. This point is further elaborated upon in the next section, where we consider a system with a simple regular maintenance schedule.

Allowing for maintenance

Maintenance can be subdivided into two categories Bedford and Cooke (2003):

Preventative: Maintenance is conducted, not because a fault or failure occurred, but as part of a regular servicing, in order to prevent failure.
Corrective: Maintenance is conducted after a fault or failure has occurred in order to restore the system.

Here we consider only preventative maintenance. As our sensitivity analysis revealed that the system failure depends most heavily on the filters, we will consider a schedule whereby these are replaced at regular time intervals. We assume that replacing the filters automatically resets these probabilities to their initial condition. We also assume that replacing the filters

only influences their probabilities of failure, without affecting the probabilities of failure of the other components. Finally, we assume that both internal and external filters are replaced at the same time.

By including maintenance of the filters at regular time intervals t_{main}, the probability of failure given by Equation 17 must be modified to

$$
\begin{aligned}
P[\text{SF}]_{\text{main}} = & C_{\text{icr}}\{F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t}) + 1 - e^{-\lambda_{\text{fs}} t_0} \\
& - [F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t})](1 - e^{-\lambda_{\text{fs}} t_0})\} \\
& + C_{\text{ecr}}(1 - e^{-\lambda_{\text{ff}} t_0}) \\
& + C_{\text{icr}} C_{\text{ecr}}\{[F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t})] \\
& (1 - e^{-\lambda_{\text{fs}} t_0})(1 - e^{-\lambda_{\text{ff}} t_0}) \\
& - [F_1 e^{-\lambda_m t} + F_2(1 - e^{-\lambda_m t})](1 - e^{-\lambda_{\text{ff}} t_0}) \\
& - (1 - e^{-\lambda_{\text{fs}} t_0})(1 - e^{-\lambda_{\text{ff}} t_0})\},
\end{aligned}
\tag{23}
$$

where t_0 is defined as

$$
t_0 = \text{mod}\left(\frac{t}{t_{\text{main}}}\right).
\tag{24}
$$

Here 'mod' denotes the remainder function, which resets the filter probabilities to their initial value every time maintenance is performed.

The probability of failure for a system under six different maintenance schedules (i.e. $t_{main} = 1$ week, 1 month, 6 months, 1 year, 2 years and 5 years) is shown in Figure 5. Regular maintenance can decrease the probability of failure significantly. However, there is an upper limit to the gains of regular maintenance of the filters, as displayed by the lower line on each of the plots. This line corresponds to the limit of $t_{main} \to 0$, i.e. infinite maintenance of the filters, and represents the
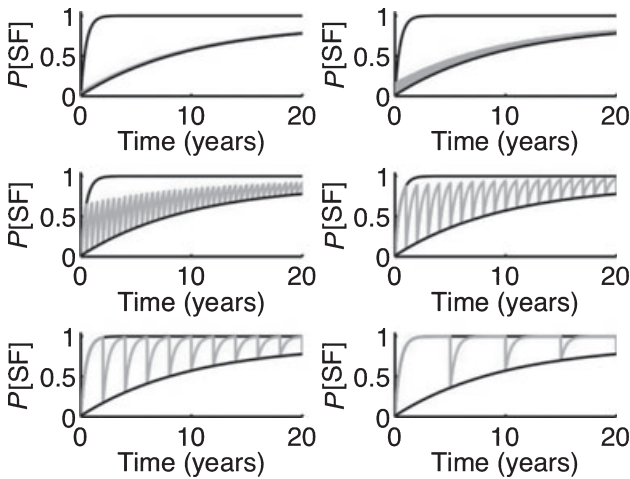

Fig. 5 Comparison of six maintenance schedules for filters: no maintenance (top dark line), maintenance (middle fluctuating light line), and infinite maintenance (bottom dark line). The parameters are set to $C_{\text{ICR}} = 1$, $C_{\text{ECR}} = 1$, $F_1 = 0.01$, $F_2 = 0.9$, $\lambda_m = 0.1$, $\lambda_{\text{fs}} = 1$ and $\lambda_{\text{ff}} = 1$

probability of failure of the system because of the other components (i.e. the mechanical ventilation system).

Which of these maintenance schedules is optimal? Clearly, the infinite maintenance case provides the best solution but is impractical. However, both the 1-week and 1-month maintenance schedules provide conditions very close to this ideal. In a practical situation one must also factor in the cost of maintenance and the particular use of the space in question. For example, if one is considering the ventilation of an operating theatre or a high-end clean room, the extra cost of maintenance is justifiable. On the other hand, if for the space in question a higher risk of contamination is acceptable, then less maintenance is required and costs can be reduced.

Limiting maintenance to the filters is likely not the optimum strategy, because as time advances with each maintenance the magnitude in the reduction of probability of failure decreases. A maintenance schedule that also considers work on the ventilation system will probably provide better returns. This is because, although the sensitivity study identifies the filters as the most critical components, the sensitivity to the mechanical ventilation system was also quite large. As mentioned previously, accounting for its maintenance requires a detailed study of the mechanical system to quantify the gain associated with each maintenance job – i.e. an additional detailed PRA study is required for the ventilation system.

For the sake of simplicity, let us assume that maintenance of the mechanical system can restore the probability of failure back to original condition (i.e. a perfect repair). Then the probability of failure of the system where the filters are maintained on a schedule t_{main} and the mechanical ventilation systems is repaired periodically every t_m years can be written as

$$
\begin{aligned}
P[\text{SF}]_{\text{main}} = & C_{\text{icr}}\{F_1 e^{-\lambda_m t_1} + F_2(1 - e^{-\lambda_m t_1}) + 1 - e^{-\lambda_{\text{fs}} t_0} \\
& - [F_1 e^{-\lambda_m t_1} + F_2(1 - e^{-\lambda_m t_1})](1 - e^{-\lambda_{\text{fs}} t_0})\} \\
& + C_{\text{ecr}}(1 - e^{-\lambda_{\text{ff}} t_0}) \\
& + C_{\text{icr}} C_{\text{ecr}}\{[F_1 e^{-\lambda_m t_1} + F_2(1 - e^{-\lambda_m t_1})] \\
& (1 - e^{-\lambda_{\text{fs}} t_0})(1 - e^{-\lambda_{\text{ff}} t_0}) \\
& - [F_1 e^{-\lambda_m t_1} + F_2(1 - e^{-\lambda_m t_1})](1 - e^{-\lambda_{\text{ff}} t_0}) \\
& - (1 - e^{-\lambda_{\text{fs}} t_0})(1 - e^{-\lambda_{\text{ff}} t_0})\},
\end{aligned}
\tag{25}
$$

where t_0 is defined in Equation 24 and t_1 is

$$
t_1 = \text{mod}\left(\frac{t}{t_m}\right).
\tag{26}
$$

Figure 6 shows how weekly maintenance of the mechanical ventilation system (the solid line) and no maintenance at all (the broken line) affect the probability of system failure. (No other component is maintained.) The two lines are virtually indistinguishable, which demonstrates that neglecting
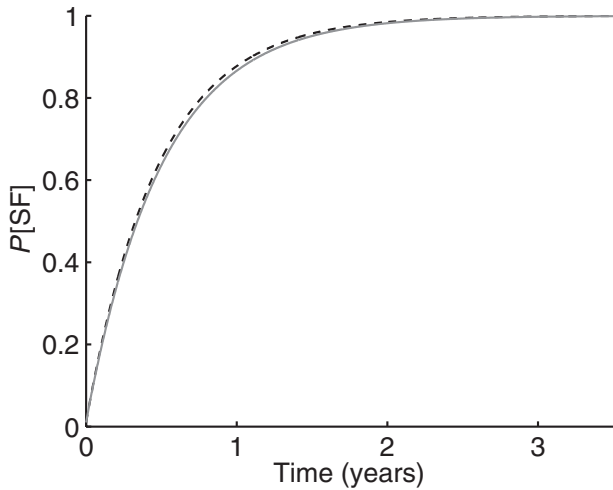
Fig. 6 Comparison of two maintenance schedules for the mechanical system: no maintenance (the broken line) and weekly maintenance (the solid line). The parameters are set to $C_{ICR} = 1$, $C_{ECR} = 1$, $F_1 = 0.01$, $F_2 = 0.9$, $\lambda_m = 0.1$, $\lambda_{fs} = 1$ and $\lambda_{ff} = 1$

maintenance on the filters is not a good approach and that the influence of solely repairing the mechanical system is negligible, no matter how often the task is performed.

Therefore, the system's performance can be optimized by maintaining both the filter and ventilation systems at properly selected time intervals. Several combinations of maintenance schedules are shown in Figure 7. The top two plots consider semi-annual maintenance of the filters combined with either weekly or every 5-year maintenance of the ventilation system. While there are some qualitative and quantitative differences between the two schedules, the overall effect does not appear large enough to justify as rigorous a
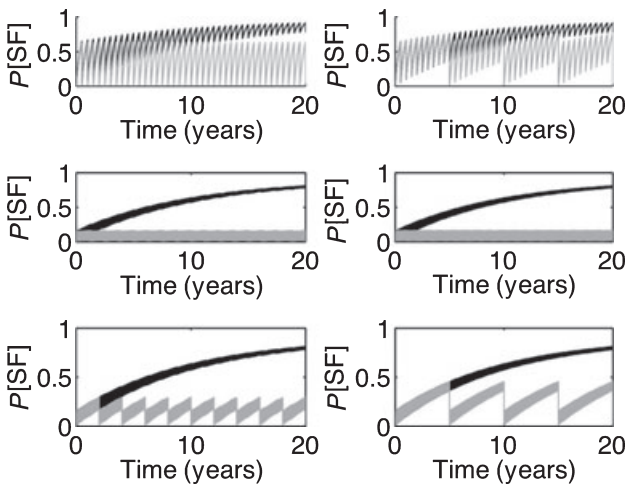

Fig. 7 Comparison of two maintenance schedules for the mechanical system: no mechanical maintenance (top black line) and mechanical maintenance (lower grey line). The parameters are set to $C_{ICR} = 1$, $C_{ECR} = 1$, $F_1 = 0.01$, $F_2 = 0.9$, $\lambda_m = 0.1$, $\lambda_{fs} = 1$, and $\lambda_{ff} = 1$

maintenance schedule as the weekly one. Again, this is because the filters affect the system failure probability most and so neglecting the filters while maintaining the mechanical system returns suboptimal gains. This leads us to consider a monthly schedule for the filters and weekly, monthly, every 2- and every 5-year maintenance of the mechanical system, which is shown in the lower four plots in Figure 7. The difference between the weekly and monthly schedule is minimal, while there is an obvious jump for the 2- and 5-year schedules. However, the 2-year schedule still yields probabilities that are not significantly higher than the monthly schedule. Once again, a balance between the desired probability of failure and the cost of maintenance is required. In order to achieve this, an optimization study that additionally considers cost should be conducted over a range of parameter space in t_{main} and t_m.

Practical approximations in PRA of ventilation

Any PRA of a ventilation system can be simplified by making certain approximations that are discussed in detail here. While the above analysis is simple and straightforward, for many practical situations these approximations become very useful.

Rare-event approximation

Let us, as an example, consider the situation where we are only concerned with internal contaminants, i.e. with the left-hand side of the fault tree in Figure 2. This is equivalent to setting $P[ECR] = 0$ or $P[FF] = 0$. The uncertainty about the basic event 'internal contaminant release (ICR)' relates not only to the occurrence of a contaminant release *per se*, but also to its precise location, strength, toxicity, duration, etc. The probability of system failure, given an internal contaminant release, can be written as

$$P[SF|ICR] = P[VF|ICR] + P[FS|ICR] - P[VF \cap FS|ICR], \tag{27}$$

where $P[X|Y]$ denotes the probability of the occurrence of X conditioned on the occurrence of Y. Suppose, for the sake of simplicity, that the release has already occurred and that all of its characteristics are known with certainty. Then $P[ICR] = 1$, and the probability of building contamination is given by

$$P[SF] = P[VF] + P[FS] - P[VF]P[FS]. \tag{28}$$

In most engineering applications, the probability of a component failure, e.g. $P[VF]$ or $P[FS]$, is typically small and the probability of a simultaneous failure of more than one components, e.g. $P[VF \cap FS]$, is often an order of magnitude smaller than that. While the latter statement is not universal and one must be cautious in applying it, it allows one to simplify Equation 28 by

employing a 'rare-event approximation' (Bedford and Cooke, 2003),

$$P[\text{SF}] \approx P[\text{VF}] + P[\text{FS}], \tag{29}$$

in which the probability of a system failure depends exclusively on the probability of failure of its constitutive parts.

Figure 8 compares the probability of system failure computed with the full solution (28) and the rare-event approximation (29). The approximation does well up to the end of first year, overpredicting the actual probability by less than 10%. This is because the assumption of a rare event becomes questionable with $P[\text{FS}] = 0.63$, which is clearly not small. Nonetheless, $P[\text{VF}] = 0.095$ can still be considered small and the error introduced by the approximation is small. Beyond this point, the approximation diverges significantly from the actual value and becomes unphysical (i.e. predicts probabilities greater than 1) after 2 years. Here the rare-event approximation completely collapses, because $P[\text{FS}] = 0.87$, which is well beyond the limit of a rare event. $P[\text{VF}] = 0.17$ is still small, but not sufficiently small as to eliminate the large influence of $P[\text{FS}]$.

The rare-event approximation (29) provides a conservative estimate of the probability of the system failure, which might prove to be overly pessimistic in many situations. When the rare-event approximation becomes invalid (e.g. if the probabilities of failure of both ventilation and filters are larger than 0.5), Equation 28 must be used instead.

Dependent probabilities – common cause approximation

In the example considered above, we assumed that all events are independent. However, the failure of the mechanical ventilation and filters could stem from a 'common cause (CC)', such as a flood damage or a power failure. In principle, any methods for uncertainty quantification should also be capable of computing not only the probabilities of basic events, such as the failures of ventilation $P[\text{VF}]$ and a filter $P[\text{FS}]$, but also the probability of their joint failure $P[\text{VF} \cap \text{FS}]$. However, for many practical implementations, this might prove to be computationally prohibitive (e.g. if a CFD analysis of the space in question is performed).

The computational burden can be reduced by identifying a CC of the failure of both the filter and ventilation. This procedure results in a conservative estimate of contamination risks and implies that the CC completely couples the occurrence of VF and FS but does not necessarily cause them,

$$P[\text{VF} \cap \text{FS}|\text{CC}] \approx P[\text{VF}|\text{CC}]. \tag{30}$$

Let CC' denote the absence of a common cause, whose probability is $P[\text{CC}'] \equiv 1 - P[\text{CC}]$. As

$$\begin{aligned}P[\text{VF} \cap \text{FS}] = &\, P[\text{VF} \cap \text{FS}|\text{CC}]P[\text{CC}] \\ &+ P[\text{VF} \cap \text{FS}|\text{CC}']P[\text{CC}'],\end{aligned} \tag{31}$$

the approximation (30) yields

$$\begin{aligned}P[\text{VF} \cap \text{FS}] \approx &\, P[\text{VF}|\text{CC}]P[\text{CC}] \\ &+ P[\text{VF} \cap \text{FS}|\text{CC}']P[\text{CC}'].\end{aligned} \tag{32}$$

Assume next that, in the absence of a CC, VF and FS are independent, $P[\text{VF} \cap \text{FS}|\text{CC}'] \approx P[\text{VF}|\text{CC}']P[\text{FS}|\text{CC}']$. Then Equation 32 becomes

$$\begin{aligned}P[\text{VF} \cap \text{FS}] \approx &\, P[\text{VF}|\text{CC}]P[\text{CC}] \\ &+ P[\text{VF}|\text{CC}']P[\text{FS}|\text{CC}']P[\text{CC}'].\end{aligned} \tag{33}$$

Finally, if $P[\text{CC}] \ll 1$ it is reasonable to assume that $P[\text{VF}|\text{CC}'] \approx P[\text{VF}]$ and $P[\text{FS}|\text{CC}'] \approx P[\text{FS}]$, so that

$$P[\text{VF} \cap \text{FS}] \approx P[\text{VF}|\text{CC}]P[\text{CC}] + P[\text{VF}]P[\text{FS}]P[\text{CC}']. \tag{34}$$

An in-depth discussion of conditional independence in Bayesian systems, which encompasses the analysis above, can be found in Pearl (2000). Expression (34) is analogous to the 'CC approximation' used in reliability analysis; chapter 8 in Bedford and Cooke (2003) provides a list of examples of common failures in engineering applications and presents a number of alternative models for dealing with them. The probability of building contamination can now be computed by combining Equations 5 and 34.

Suppose that a probabilistic analysis of a model and site-characterization data resulted in the following (subjective) probabilities: the probability that a contaminant exceeds some threshold value at time $t = T$ through a CC is $P[\text{CC}] = 0.01$. If the contamination is due to the common cause, the probabilities of failure of
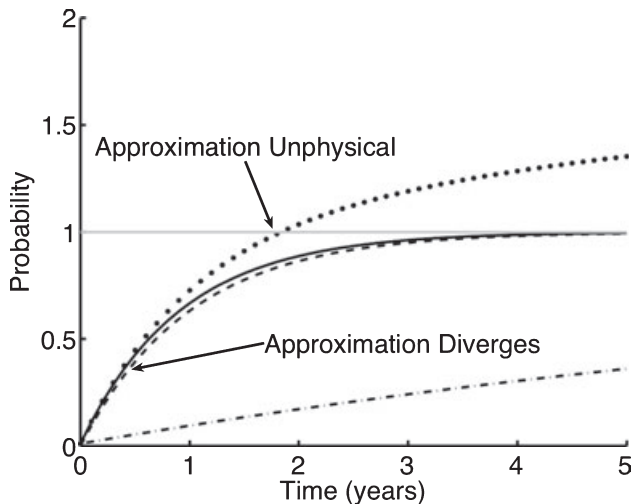

Fig. 8 Probability of system failure computed with the full solution (28) (the solid line) and the rare-event approximation (29) (the dotted line). Also shown in this figure are the probabilities of failure of ventilation, $P[\text{VF}]$ (–·), and filters, $P[\text{FS}]$ (- -)

both ventilation and the filter at time $t = T$ are $P[VF|CC] = P[FS|CC] = 1$. Practically speaking, an example of such a CC could be a power failure or flooding.

The probability of building contamination at time $t = T$ computed with the rare-event approximation (29) is $P[SF] = 0.6$, while its counterpart computed with the CC approximations (5) and (34) is $P[SF] = 0.54$. Note that the contribution of the low-probability common cause PF to both the probability of the joint failure of ventilation and filtration and the probability of building contamination is significant. Another interesting observation is that the rare-event approximation (29) gives a reasonably accurate risk estimate, even though the probability of failure of ventilation failure is 50%. This simple analysis can be used to obtain rough estimates of the risks of building contamination. A more detailed and rigorous PRA could incorporate the use of the probabilistic and stochastic tools.

Figure 9 displays the influence of the CC approximation. For a very rare CC, e.g. $P[CC] = 0.01$, the influence of the CC on the probability of failure is very small, as one would expect. However, the influence becomes larger as the probability of the CC increases, leading to differences that could be considered large depending on the particular application. Neglecting the influence of a CC can be thought of as a worst-case scenario. As such, for spaces that require particularly low probabilities of contamination, it could be neglected, which adds an additional factor of safety.

Summary

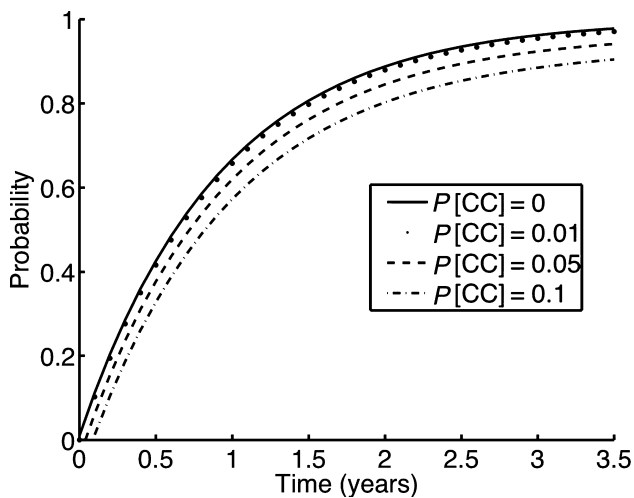We introduced PRA as a tool that could be used to quantify contamination risks in ventilated buildings.

Fig. 9 Influence of a common cause (CC) on the probability of failure. $P[CC] = 0$ (–), $P[CC] = 0.01$ (··), $P[CC] = 0.05$ (- -), and $P[CC]$ (–·)

The general PRA framework is capable of handling phenomena that range from contamination of residential houses to large office buildings to clean-room manufacturing and operating theaters and could easily be extended to other areas of interest, such as contaminants in multiply connected spaces (e.g. hospitals, where there are many rooms where contaminants are prevalent, and others where it is essential to maintain a clean environment). This framework can be used to make decisions under uncertainty, including: (a) determination of the viability of the ventilation system and other alternative remediation strategies and (b) optimization of data collection and monitoring campaigns. Key features of this approach are: (i) the comprehensive treatment of structural (model) and parametric uncertainties inherent in building flows and contaminant transport and (ii) the use of subjective probabilities, i.e. the reliance on expert knowledge.

Further information that would be useful in a comprehensive PRA can be obtained by making it problem-specific. For example, a wide range of contamination problems do not necessarily require point-wise prediction of contaminant behavior (e.g. an office space where standards are based on average concentrations). For such a case, mass-balance calculations are often sufficient to assess probabilities of the occurrence of basic events by quantifying uncertainty in appropriate lumped-parameter models, i.e. by combining PRA with the results from lumped-parameter models developed in Haghighat et al. (1988), Hunt and Kaye (2006), Nazaroff and Cass (1991), etc. Other problems may require detailed knowledge of the contaminant distribution (e.g. an operating theater where the sole focus is to keep patients contaminant-free). For such scenarios, CFD studies or other distributed (non-lumped parameter) models (Bolster and Linden, 2007) should be used.

Lumped-parameter models yield simple, closed-form expressions for the bulk behavior of a contaminant and its response to various remediation strategies. A typical example of such analyses is provided by Nazaroff and Cass (1991), who modeled the spread of particulate contaminants in art museums and the spread of tobacco fumes in occupied spaces. Their analytical solutions depend on a number of parameters, some of which are measured (e.g. flow rates) and some are fitted to data (e.g. particle-settling coefficient). In any field application, the values of these parameters can be uncertain and probabilistic methods could be used.

Any PRA of ventilation processes must be flexible and extensible enough to make optimal use of existing site characterization data and to accommodate new information, including new data and conceptual models. The extensibility is critical for both the long-term relevance of the framework and its impact on the development and deployment of effective tools for

monitoring and/or remediation of contaminated sites. It can be achieved by using some of the appropriate probabilistic tools for quantification of various types and levels of uncertainty that contribute to the overall predictive uncertainty. The flexibility comes from the modular use of some or all of these techniques and the possibility of incorporating other approaches. Applications of the PRA approach to complex ventilation problems might necessitate a computerized construction of fault trees; and commercially PRA software is widely available.

The work presented herein is not meant to be a comprehensive study of contamination of buildings and all associated risks, but merely an illustration of an as yet untapped resource available to the field of building contamination. A real building system can be far more complicated with many additional factors that could be considered, if necessary, in an appropriate analysis. Additionally, further development of the knowledge on appropriate probabilities and proper empirical verification are required before it is suggested as a standard tool for the practitioner.

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