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Particle transport in low-energy ventilation systems. Part 1: theory of steady states

Abstract Many modern low-energy ventilation schemes, such as displacement or natural ventilation, take advantage of temperature stratification in a space, extracting the warmest air from the top of the room. The adoption of these energy-efficient ventilation systems still requires the provision of acceptable indoor air quality. In this work we study the steady state transport of particulate contaminants in a displacement-ventilated space. Representing heat sources as ideal sources of buoyancy, analytical models are developed that allow us to compare the average efficiency of contaminant removal between traditional and modern low-energy systems. We found that on average traditional and low-energy systems are similar in overall pollutant removal efficiency, although quite different vertical distributions of contaminant can exist, thus affecting individual exposure. While the main focus of this work is on particles where the dominant mode of deposition is by gravitational settling, we also discuss additional deposition mechanisms and show that the qualitative observations we make carry over to cases where such mechanisms must be included.

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Practical Implications

We illustrate that while average concentration of particles for traditional mixing systems and low energy displacement systems are similar, local concentrations can vary significantly with displacement systems. Depending on the source of the particles this can be better or worse in terms of occupant exposure and engineers should take due diligence accordingly when designing ventilation systems.

Introduction

We live in world where 'energy consumption defines the quality of urban life' (Santamouris, 2005). Developed countries consume massive amounts of energy while only accounting for a small fraction of the global population. According to the Energy Information Administration (http://www.eia.doe.gov/) the US alone produces 25% of the world's total anthropogenic CO_2, while accounting for $<5\%$ of the world's population. A major fraction is produced by modern buildings, which consume approximately 40% of the world's energy and are responsible for 50% of global anthropogenic CO_2 emissions. A significant fraction of this energy is spent on ventilation of buildings with summer time cooling account for almost 10% of the US total energy budget.

To reduce energy consumption various low-energy systems such as displacement-ventilation, underfloor air distribution, operable windows, night cooling, radiant and evaporative cooling are under consideration. Underlying all these systems is the idea that free cooling is possible. Traditional ventilation, such as that provided by a conventional overhead heating, ventilating and air-conditioning system, is mixing ventilation, where incoming air is mixed with the air in the room and diluted. This typically results in a relatively uniform interior temperature distribution. In contrast, many modern low-energy ventilation schemes require the use of temperature stratification in a space, with a bottom layer of cooler comfortable air where occupants are located, and an unoccupied upper layer that is comparatively warm (Linden, 1999). The ability to extract air at elevated temperatures is necessary for energy-efficiency and free cooling. This can be achieved, for example, by displacement-ventilation, underfloor air distribution or natural ventilation. Hence, stratification is an important feature in modern ventilation design. This is particularly true for tall spaces, where temperature differences can be quite significant.

People spend substantial amounts of time indoors, in many cases up to as much as 90% (Jenkins et al., 1992) and, therefore, the provision of a thermally comfortable environment in an energy-efficient manner is only one requirement of a ventilation system. It is also important to understand the details of the indoor environment regarding indoor air quality (IAQ). It is often stated that such displacement-ventilation systems can be more effective at removing contaminants (e.g. Brohus and Nielsen, 1996; Lin et al., 2005; Xinga et al., 2001). In a previous study (Bolster and Linden, 2007), the authors showed that this may in fact not always be true and that the average rate of contaminant removal for passive contaminants is quite similar for traditional and modern low-energy systems. Additionally, experimental studies (Mundt, 2001) have shown that the ventilation effectiveness of a displacement system is sensitive the location and type of the contaminant source involved. Displacement systems typically result in different vertical concentration gradients and in some cases can lead to larger exposure of occupants to contaminants.

While the study of passive tracer contaminants is important in understanding ventilation system efficiency, there is another type of contaminant that must also be considered - particulates. The concentration and composition of indoor particles are major determinants of environmental quality, as inhalation exposure poses potentially adverse effects on people's health. Such particles can penetrate into buildings from the outdoors or can be emitted from indoor sources such as smoking, cooking, occupants, building materials or even during a deliberate malicious release. The study of particulate matter is more complicated than that of passive contaminants. With particles there are various other phenomena that can occur – deposition, resuspension, coagulation, and filtration, which are all difficult to model and quantify. In particular, gravitational settling raises the concern that particles may not be removed as efficiently from a system that is extracting air from the top of a room, which is typical of low-energy ventilation systems.

Mathematical models

To understand the fate of particles in a ventilated space, it is necessary to understand the flow within the space. As mentioned previously, many modern low-energy ventilation schemes, such as displacement or natural ventilation, exploit vertical temperature stratification in a space. So it is critical to understand how heat sources within a ventilated enclosure stratify that space. Many heat sources within a building can be regarded as localized and can often be modeled as pure sources of buoyancy. Using the plume equations developed by Morton et al., 1956 along with the 'filling' (Baines and Turner, 1969) and 'emptying-filling' box models (Linden et al., 1990) we can model the flow in such low energy buildings, and calculate the transport of particulate contaminants within the interior space.

Figure 1 shows a schematic of the two models that we consider. We analyze one low-energy ventilation model (b) and one traditional mixing model (a). In the low-energy models we consider the space either mechanically or naturally ventilated with fresh air entering through a low level vent and hot buoyant air leaving via a vent at high level. Heat sources in the space are represented by ideal plumes. As we are only considering steady states in this particular paper, we are not concerned with the detailed vertical contaminant distribution in the two layer model (see Bolster and Linden, 2007) as the simplest model with two well mixed layers results in the same steady state as other two layer models. Figure 2 illustrates the transport processes for the contaminant for both the well mixed and two-layer cases. In part 2 of this work we consider a model where vertical gradients of contaminant can exist.

Contamination scenarios considered

For each of the models depicted in Figure 1, we consider two types of contamination situations:

1. *External contaminant (Step up):* Here we consider a case where contaminant is introduced in through the ventilation system. This can also be thought of as a contaminant that is located in the lower layer of the two-layer ventilation system. In terms of Figure 2 this corresponds to K_{in} being non-zero and $K_s = 0$.
2. *Internal contaminant (Isolated source in plume):* Here contaminant is introduced as a point source located within the plume. We choose this location because people are often the source of heat as well as the source of contaminants in buildings. This can also be thought of as a contaminant located in the upper layer of the displacement-ventilation system. In terms of Figure 2 this corresponds to K_s being non-zero and $K_{in} = 0$.

Model (a) - entirely well mixed space

In this model we treat the entire room as well mixed (Figure 1a). The reason for this is twofold. First, it allows us to compare low-energy ventilation systems to traditional mixing systems, which minimize stratification by mixing the space. Second, many building software packages treat buildings as networks of spaces that are each assumed to be well mixed. As many

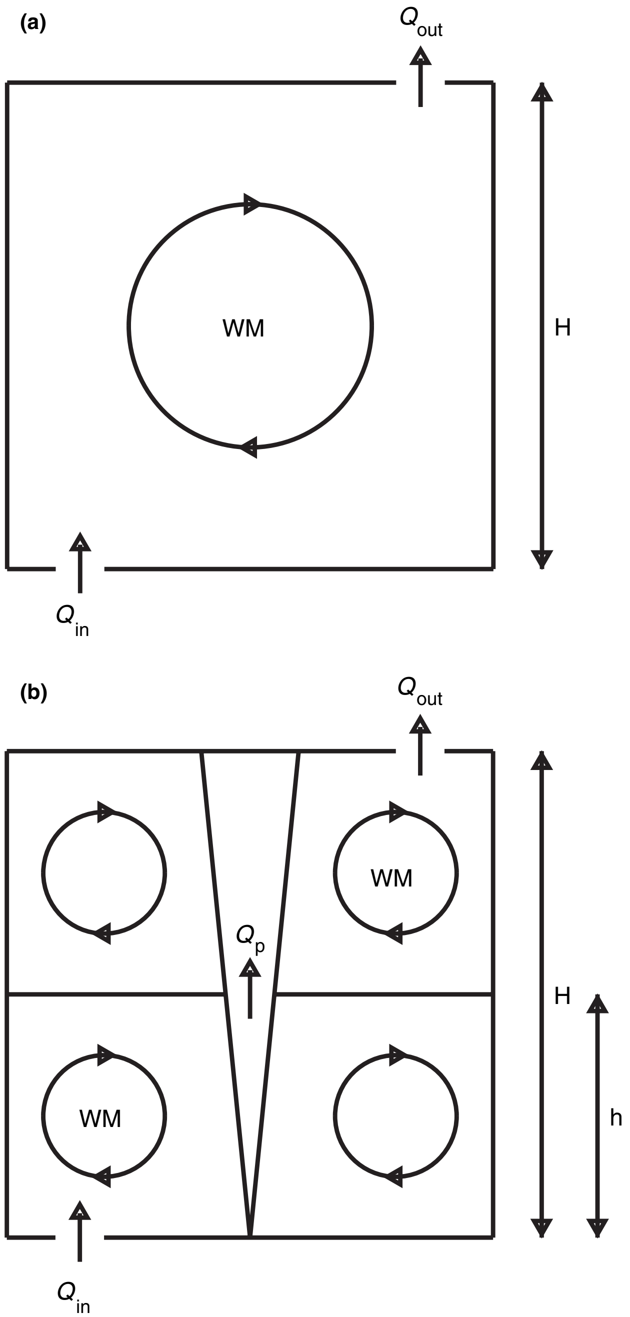

Fig. 1 Models of displacement-ventilation systems. (a) single well mixed layer, (b) two-layer system

researchers have previously pointed out (e.g. Baughman et al., 1994; Klepeis, 1999), this assumption is questionable and so we test it here. At steady state the concentration in a well mixed space, K_{wm}, satisfies the conservation equation

$$(Q_{out} + Q_{fall})K_{wm} = Q_{in}K_{in} + Q_s K_s, \quad (1)$$

where Q_{in} is the flow rate into the space, Q_{out} is the flow rate into the space, Q_s is the flow rate associated with an internal contaminant source, $Q_{fall} = v_{fall}\ S$ is defined as the settling flow rate, v_{fall} is the settling velocity, assumed here to be the Stokes settling velocity, K_{in} is the concentration of contaminant in the incoming air, K_s is the concentration associated with a point source (defined in greater detail in appendix A), S is the room cross sectional area and H is the height of the room. As outlined in appendix A we take $Q_s = Q_p$, where Q_p is the volume flux associated with the plume across the interface for the two-layer case. At steady state $Q_{in} = Q_{out} = Q_p$ (see Linden et al., 1990). Q_{fall} quantifies the amount of deposition that will take place. We neglect deposition of particles to the ceiling and sidewalls and assume that particles settle out of the lower and upper layers at this settling velocity. This is a reasonable assumption for larger particles ($> O(0.1–1)\mu$m), where the predominant mechanism of deposition is gravitational settling and Brownian effects are negligible (Lai and Nazaroff, 2000). For ultrafine particles ($< O(0.1–1)\mu$m) deposition will also be driven by Brownian effects. Deposition due to Brownian effects is strongly dependent on the turbulent friction velocity at the boundaries of the room. As for displacement-ventilation, characteristic velocities are typically an order of magnitude smaller than for traditional mixing systems (Jiang et al.,1992), it is reasonable to assume that deposition effects driven by Brownian settling will also be much smaller and only become significant for smaller particles ($< O(0.1)\mu$m).

Model (b) - well mixed two layer model

In this section we consider model (b) from Figure 1. We take an approach similar to that of Hunt and Kaye (2006) and assume that the upper and lower layers are always well mixed. The justification for this assumption is that the plume will cause some mixing in the upper layer. However, in a previous study on passive contaminants (Bolster and Linden, 2007) we found that this assumption does not describe the complete dynamics of the system. None the less, at least for passive contaminants, it has been shown to be an adequate model (Hunt and Kaye, 2006) and is very appealing because of its simplicity. We also assume that the lower layer is well mixed. As the incoming flow will have a finite amount of momentum, a certain amount of mixing will be inevitable and in our previous work on passive contaminants we showed that this is a reasonable assumption. Thus the governing equations for conservation of contaminant in each of the layers are

$$(Q_p + Q_{fall})K_l = Q_{fall}K_u + Q_{in}K_{in}$$

$$(Q_{fall} + Q_p)K_u = Q_p K_l + Q_p K_s \quad (2)$$

where K_l and K_u are the concentrations of contaminant in the lower and upper layers, respectively, h is the height of the lower layer and Q_p is the plume flow rate across the interface and at steady state $Q_p = Q_{in}$.

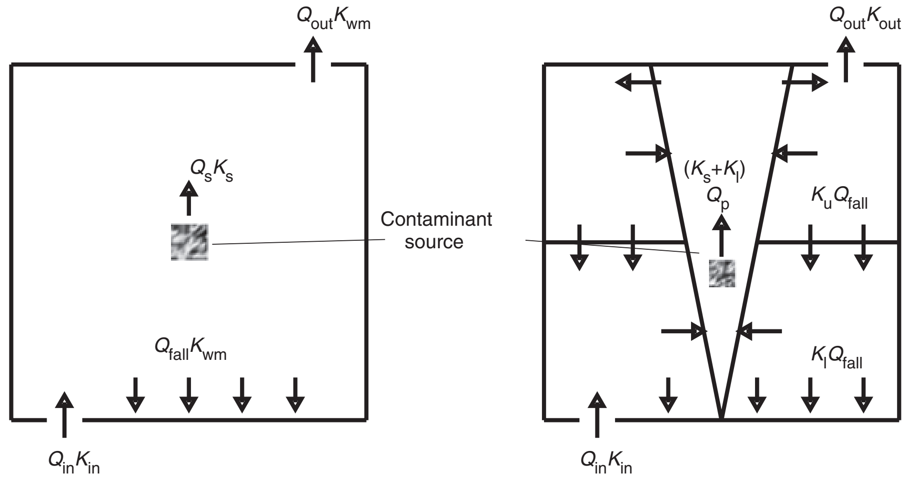

Fig. 2 A schematic illustrating the contaminant transport processes taking place in the well mixed and two-layer cases

Non-dimensionalization

We non-dimensionalize as follows:

$$K = K_{\mathrm{ref}}\kappa, \qquad h = H\zeta, \tag{3}$$

where K_{ref} is a reference concentration, which will be different for each of the three situations considered. For the step-up system it is the concentration of contaminant entering the spaces ($K_{\mathrm{ref}} = K_{\mathrm{in}}$) and for the point source case it is the concentration of the source ($K_{\mathrm{ref}} = K_{\mathrm{s}}$). This results in the following dimensionless equations:

(a)

$$(1+\alpha)\kappa = \kappa_{\mathrm{in}} + \kappa_{\mathrm{s}}, \tag{4}$$

(b)

$$(1+\alpha)\kappa_{\mathrm{l}} = \alpha\kappa_{\mathrm{u}} + \kappa_{\mathrm{in}},$$

$$(1+\alpha)\kappa_{\mathrm{u}} = \kappa_{\mathrm{l}} + \kappa_{\mathrm{s}}, \tag{5}$$

where $\alpha = \frac{Q_{\mathrm{fall}}}{Q_{\mathrm{p}}}$, which is a dimensionless representation of the particle settling velocity.

Results

External contaminant

We consider the situation where contaminant is introduced via the ventilation system. This can correspond to a number of scenarios, such as a leak in a ventilation system, a malicious release, or an external contaminant entering the building though natural ventilation. Here $\kappa_{\mathrm{in}} = 1$ and $\kappa_{\mathrm{s}} = 0$, which should be substituted in to the equations presented in the previous section. For a passive contaminant this steady state corresponds to a uniformly distributed concentration of contaminant equal to that of the source. However, the influence of gravitational settling leads to nontrivial steady state distributions.

From (4) we predict that the well mixed space in model (a) tends to a uniform contaminant concentration of

$$\kappa^{(a)} = \frac{1}{1+\alpha}. \tag{6}$$

This is lower than the concentration of fluid entering the space, because there is an additional sink in the deposition term that does not extract fluid, but does extract contaminant. For the two-layer case it can be shown from (5) that for such a system

$$\kappa_{\mathrm{u}}^{(b)} = \frac{1}{(\alpha+1)^2-\alpha}, \qquad \kappa_{\mathrm{l}}^{(b)} = \frac{(1+\alpha)}{(\alpha+1)^2-\alpha}. \tag{7}$$

Therefore, at steady state, the concentration in the lower layer is always greater than that in the upper layer and occupants, assumed to be located in the lower layer, are exposed to the highest concentrations in the space. Interestingly, this steady state is also independent of ζ, the interface height.

We can compare the steady state value of the concentration of lower layer for model (b) to the well mixed case, which results in

$$\frac{\kappa_{\mathrm{l}}^{(b)}}{\kappa^{(a)}} = \frac{(1+\alpha)^2}{(1+\alpha)^2-\alpha} > 1. \tag{8}$$

This ratio is also independent of ζ the interface height. It is plotted in Figure 3(b). Additionally, regardless of the value of α the lower layer always has a higher level of contaminant than the well mixed case. Thus people are always exposed to a higher concentration in the low-energy ventilation system. It is worth noting that there is a maximum value for the ratio $\frac{\kappa_l^{(b)}}{\kappa^{(a)}} = 1.33$ at $\alpha = 1$, which means that this corresponds to the worst case scenario regarding a comparison between traditional and low-energy ventilation systems.

However, if we only consider the overall average concentration at steady state of model (a) vs. model (b) we find

$$\frac{\bar{\kappa}^{(b)}}{\bar{\kappa}^{(a)}} = \frac{\zeta\kappa_l + (1-\zeta)\kappa u}{\frac{1}{1+\alpha}} = \frac{(1+\zeta\alpha)(1+\alpha)}{\alpha^2+\alpha+1}. \qquad (9)$$

Figure 3 (a) depicts the ranges of α and ζ where the average concentration for the traditional mixing system is higher than the low-energy two-layer systems.

Internal source

As for the external case, the ultimate steady state that the system reaches will differ for the traditional and low energy systems. Here $\kappa_{in} = 0$ and $\kappa_s = 1$, which should be substituted in to the equations presented in section 3.

The well mixed space in model (a) tends to a uniform contaminant concentration of (this is calculated by assuming a source of κ_s in the space as is perceived in the upper layer equation above - see appendix A for details)

$$\kappa^{(a)} = \frac{1}{1+\alpha}. \qquad (10)$$

This is lower than the concentration of fluid entering the space, because of the sink effect of deposition that does not extract fluid, but does extract contaminant.

For the two-layer cases both systems tend to the same steady state where both layers are well mixed. For this situation the upper and lower layer concentration fields are

$$\kappa_u^{(b)} = \frac{1+\alpha}{(\alpha+1)^2-\alpha}, \qquad \kappa_l^{(b)} = \frac{\alpha}{(\alpha+1)^2-\alpha} \qquad (11)$$

respectively. At steady state, the concentration in the upper layer is always greater than that in the lower layer and people, who only occupy the lower layer, are only exposed to the lowest concentrations in the space. Again, these steady state values are independent of ζ.

Comparing the concentration of the lower layer for models (b) and (a) we obtain

$$\frac{\kappa_l^{(b)}}{\kappa^{(a)}} = \frac{\alpha^2+\alpha}{\alpha^2+\alpha+1} < 1, \qquad (12)$$

which indicates that for this type of point source the low-energy system always does a better job removing contaminants than the traditional system, regardless of the interface location or particle size. This ratio is zero for $\alpha = 0$, which corresponds to a passive contaminant, and approaches 1 as $\alpha \to \infty$. This is reasonable because the source is effectively in the upper layer and for $\alpha = 0$, no contaminant can fall back into the lower layer. However, as α increases, more contaminant can fall through, thus increasing the concentration of the lower layer.

On the other hand, if we only consider the average contaminant removal, we can see from Figure 4 that there are regions, particularly as the particle size increases, where the two-layer system is worse (grey region) at removing contaminants than the one-layer well-mixed system. However, since from a practical perspective we only care about concentrations in the lower layer, this is not really the point of interest and is merely shown here to illustrate that an average contaminant concentration value is deceptive in predicting an individual's exposure as illustrated in the experiments by Ozkaynak et al. (1982).

Additional mechanisms of deposition

While gravitational effects dominate the deposition mechanisms for large particles (typically $> 1\mu$m, although this is dependent on the friction velocity at a boundary, which for a displacement system should be less than for traditional mixing system), the deposition of particles smaller than this can be strongly driven by Brownian diffusion (Lai and Nazaroff, 2000). Therefore, for such particles the governing equations (4)–(5)

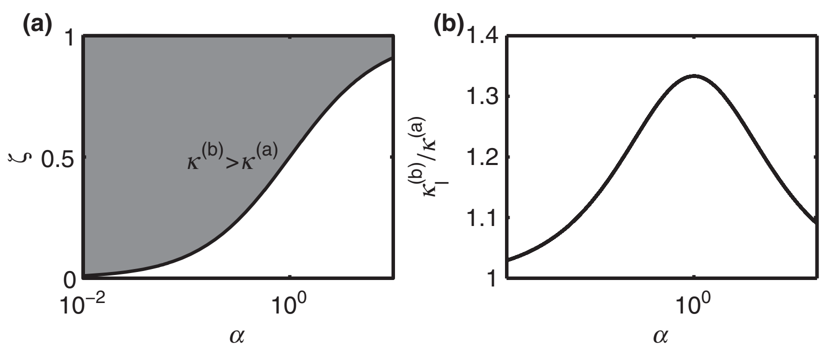

Fig. 3 (a) Comparison of the steady state average concentration across the entire height of the space for the single layer vs. two layer models (b) Ratio of the steady state concentrations of the lower layer in the two layer models to the single layer concentration ($\kappa^{(b)}/\kappa^{(a)} > 1$)

must be modified to account for this. As the ultimate steady state for both two layer models is the same we focus on model (b) here. Accounting for additional settling to all surfaces the governing equations become

(a)

$$(Q_{\text{in}} + Q_{\text{dw}})K_{\text{wm}} = Q_{\text{in}}K_{\text{in}} + Q_{\text{in}}K_{\text{s}}, \tag{13}$$

(b)

$$(Q_{\text{p}} + Q_{\text{dl}})K_{\text{l}} = Q_{\text{df}}K_{\text{u}} + Q_{\text{in}}K_{\text{in}}$$

$$(Q_{\text{du}} + Q_{\text{p}})K_{u} = (Q_{\text{p}} + Q_{\text{dr}})K_{\text{l}} + Q_{\text{p}}K_{\text{s}} \tag{14}$$

where Q_{dw} is the flow rate at which particles settle out of the well mixed space, Q_{dl} is the flow rate at which particles settle out of the lower layer, Q_{df} is the flow rate of particles that flow from the upper to lower layer across the interface, Q_{du} is the flow rate at which particles settle out the upper layer and Q_{dr} is the flow rate at which particles cross the interface from the lower to upper layers. These quantities are evaluated as follows:

$$Q_{\text{dw}} = v_{\text{v}}A_{\text{v}} + v_{\text{d}}A_{\text{d}} + v_{\text{u}}A_{\text{u}} \quad Q_{\text{dl}} = v_{\text{v}}A_{\text{v}}^{l} + v_{\text{d}}A_{\text{d}} + v_{\text{u}}A_{\text{u}} \quad Q_{\text{df}} = v_{\text{u}}A_{\text{u}}$$

$$Q_{\text{du}} = v_{\text{v}}A_{\text{v}}^{u} + v_{\text{d}}A_{\text{d}} + v_{\text{u}}A_{\text{u}} \quad Q_{\text{dr}} = v_{\text{d}}A_{\text{d}} \tag{15}$$

where v_{v} is the deposition velocity of a particle depositing on to a vertical surface, v_{d} is the deposition velocity of a particle depositing on to a downward facing horizontal surface, v_{u} is the deposition velocity of a particle depositing on to an upward facing horizontal surface, A_{v} is the total area of vertical boundaries in the space, A_{v}^{l} is the area of vertical boundaries in the lower layer, A_{v}^{u} is the area of vertical boundaries in the upper layer, A_{u} is the area of the an upward facing boundary and A_{l} is the area of downward facing boundaries. The deposition velocities can be evaluated using equations presented in Table 2 in Lai and Nazaroff (2000). We treat the interface in the two layer model as a 'fictitious' rigid boundary through which fluxes can occur.

In dimensionless terms (13) and (14) become:

(a)

$$(1 + \alpha_{\text{dw}})\kappa = \kappa_{\text{in}} + \kappa_{\text{s}}, \tag{16}$$

(b)

$$(1 + \alpha_{\text{dl}})\kappa_{\text{l}} = \alpha_{\text{df}}\kappa_{\text{u}} + \kappa_{\text{in}},$$

$$(1 + \alpha_{\text{du}})\kappa_{\text{u}} = (1 + \alpha_{\text{dr}})\kappa_{\text{l}} + \kappa_{\text{s}}, \tag{17}$$

where $\alpha_{\text{di}} = \frac{Q_{\text{di}}}{Q_{\text{in}}}$ represents the dimensionless forms of the various deposition flow rates defined in (15). The subscript i can represent the subscripts w, l, f, u or r. By accounting for these additional mechanisms we introduce several new dimensionless parameters. In the limit of large particles, the deposition velocities to upward facing surfaces reduces to the settling velocity, while the deposition to downward facing and vertical surfaces reduces to zero and we recover the equation presented in sections 2–4. The steady state concentrations for each of the models are given by

$$\kappa^{(a)} = \frac{\kappa_{\text{in}} + \kappa_{\text{s}}}{1 + \alpha_{\text{dw}}}, \tag{18}$$

$$\kappa_{\text{l}}^{(b)} = \frac{(1 + \alpha_{\text{du}})\kappa_{\text{in}} + \alpha_{\text{df}}\kappa_{\text{s}}}{1 + \alpha_{\text{dl}} + \alpha_{\text{du}}(1 + \alpha_{\text{dl}}) - \alpha_{\text{df}}(1 + \alpha_{\text{dr}})} \tag{19}$$

$$\kappa_{\text{u}}^{(b)} = \frac{(1 + \alpha_{\text{dr}})\kappa_{\text{in}} + (1 + \alpha_{\text{dl}})\kappa_{\text{s}}}{1 + \alpha_{\text{dl}} + \alpha_{\text{du}}(1 + \alpha_{\text{dl}}) - \alpha_{\text{df}}(1 + \alpha_{\text{dr}})}. \tag{20}$$

External contaminant case. For the external contaminant situation we considered previously (i.e. $\kappa_{\text{in}} = 1$ and $\kappa_{\text{s}} = 0$) we again compare the upper to lower layer concentrations in the two-layer system. We also compare the lower layer concentration in the two-layer

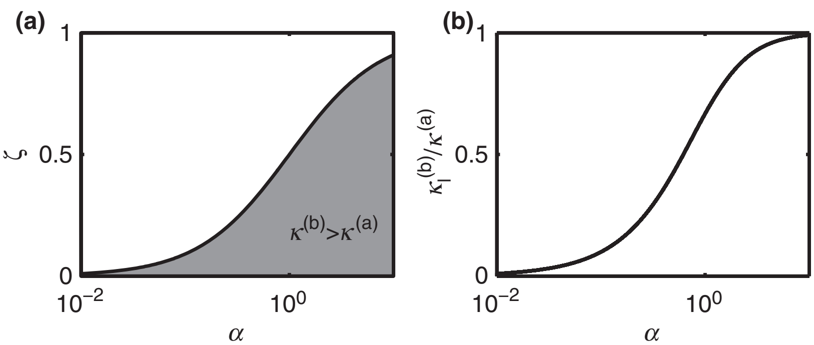

Fig. 4 (a) Comparison of the steady state average concentration across the entire height of the space for the single layer vs. two layer models (b) Ratio of the steady state concentrations of the lower layer in the two layer models to the single layer concentration ($\kappa^{(b)}/\kappa^{(a)} < 1$)

system to the concentration in the traditional well mixed space.

$$\frac{\kappa_{\mathrm{l}}^{(b)}}{\kappa_{\mathrm{u}}^{(b)}} = \frac{1+\alpha_{\mathrm{du}}}{1+\alpha_{\mathrm{dr}}}. \tag{21}$$

Because α_{du} includes deposition to vertical and horizontal surfaces, while α_{dr} only involves deposition to a downward facing horizontal surface, it is readily seen that $\alpha_{\mathrm{du}} > \alpha_{\mathrm{dr}}$ and therefore $\kappa_l^{(b,c)} > \kappa_{\mathrm{u}}^{(b,c)}$. Once again for the step-up case the concentration in the lower layer is always greater than it is in the upper layer, even with the additional settling mechanisms for fine and ultrafine particles. Now we compare the lower layer concentration in the two-layer system to the concentration in the traditional mixing space

$$\frac{\kappa_{\mathrm{l}}^{(b)}}{\kappa^{(a)}} = \frac{(1+\alpha_{\mathrm{du}})(1+\alpha_{\mathrm{dw}})}{1+\alpha_{\mathrm{dl}}+\alpha_{\mathrm{du}}(1+\alpha_{\mathrm{dl}})-\alpha_{\mathrm{df}}(1+\alpha_{\mathrm{dr}})}. \tag{22}$$

It is relatively straightforward using (15) to show to that the denominator is greater than the numerator in (22). Therefore, as we observed previously, occupants are exposed to higher levels of contaminants in the low energy system when a step-up case is considered.

Internal source case. In the same manner we can consider the internal source situation ($\kappa_{\mathrm{in}} = 0$ and $\kappa_{\mathrm{s}} = 1$), where

$$\frac{\kappa_{\mathrm{l}}^{(b)}}{\kappa_{\mathrm{u}}^{(b,c)}} = \frac{\alpha_{\mathrm{df}}}{1+\alpha_{\mathrm{dl}}}. \tag{23}$$

Now, from (15), we know that $\alpha_{\mathrm{df}} < \alpha_{\mathrm{dl}}$ Therefore, the lower layer concentration is always less than that in the upper layer. Similarly

$$\frac{\kappa_{\mathrm{l}}^{(b)}}{\kappa^{(a)}} = \frac{\alpha_{\mathrm{df}}(1+\alpha_{\mathrm{dw}})}{1+\alpha_{\mathrm{dl}}+\alpha_{\mathrm{du}}(1+\alpha_{\mathrm{dl}})-\alpha_{\mathrm{df}}(1+\alpha_{\mathrm{dr}})}. \tag{24}$$

Once again using (15), we can show that the denominator is less than the numerator in (24). Therefore, as we observed previously, occupants are exposed to lower levels of contaminants in the low energy system when a point source is considered.

Conclusions

In this paper we have considered the steady transport of particulate contaminants in a displacement-ventilated space. We compared two models, one representing a traditional ventilation system and the other representing a displacement-ventilated space. We considered two contamination scenarios, namely an external and an internal contaminant source. Several important differences between the traditional and low-energy systems were noted.

It is widely believed that low-energy displacement-ventilation systems can be better than traditional mixing systems at removing contaminants from a space. This is because there is a belief that these systems will use the same mechanism for contaminant removal as they do for heat removal, where they are clearly more efficient. The heat extraction problem exploits the natural stratification that develops, extracting the warmest air that naturally sits at the top of the room. However, there is no physical justification as to why this location should correspond to the location of maximum contaminant concentration too. In fact many times it does not (Bolster and Linden, 2007).

For the external contaminant case, we showed that, at steady state, the concentration in the lower layer is greater than that of the upper layer. Further, this lower layer concentration is larger than that for an equivalent traditional ventilation system. The largest difference occurs for particles with $\alpha = 1$, where the lower layer concentration in the displacement system is 33% higher than that in the traditional system.

On the other hand, when considering the internal contaminant scenario, we predict a higher steady state concentration in the upper layer compared to the lower layer. The lower layer concentration will always be less than that in an equivalent traditional system, thus reducing occupants' exposure to contaminants.

It is clearly important to consider the types of sources that are likely to be encountered in a real building. For example, in a well designed surgical operating theater, the ventilation system typically filters out most contaminant before introducing air into the room. Therefore, it is unlikely that the step-up scenario is relevant. In an operating theater the most common sources of contaminants are the surgeons, nurses and patients (Smith, 1975), which would correspond to the point source problem described herein. As such, a displacement system may provide better air quality than a traditional mixing ventilation system. On the other hand, if we consider a naturally ventilated space, where external sources can play an important part in contamination, the step-up scenario may be relevant.

Another important point to note is that it is not always sufficient to estimate the average amount of contaminant within a space. As shown for all three contamination scenarios, computing the average concentrations only can lead to an overly optimistic picture as local concentrations can often be significantly higher. In many cases one ventilation system can outperform another based on average concentrations. However, when considering individual exposure this may no longer hold true.

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Appendix A. point source strength estimation

A point source in the plume in the lower layer can be thought of as an additional source into the upper layer.

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \frac{5}{3z}(K - P), \qquad z < h \tag{A.1}$$

which implies that if we have a source of strength $\hat{K}_s$ at a height z_s in the lower layer, the concentration being injected into the upper layer by the plume is

$$P(z = h) = K_l + \hat{K}_s\left(\frac{z_s}{h}\right)^{\frac{5}{3}} = K_l + K_s \tag{A.2}$$

Therefore the conservation equations can be written as

$$\frac{\mathrm{d}K_l}{\mathrm{d}t} = -\left(\frac{Q_p + Q_{fall}}{Sh}\right)K_l + \frac{Q_{fall}}{Sh}K_u + \frac{Q_{in}}{Sh}K_{in}, \tag{A.3}$$

$$\frac{\mathrm{d}K_u}{\mathrm{d}t} = \frac{Q_p}{S(H-h)}(K_l + K_s) - \left(\frac{Q_{fall} + Q_p}{S(H-h)}\right)K_u. \tag{A.4}$$

To compare equivalent systems the conservation equation for the well mixed room should include a source of the same strength leading to

$$\frac{\mathrm{d}K_{wm}}{\mathrm{d}t} = \frac{Q_{in}(K_{in} + K_s)}{SH} - \frac{Q_{in} + Q_{fall}}{SH}. \tag{A.5}$$