Open Channel Flow
Part 2

Ch 10 Young, notes, handouts
Uniform Channel Flow

• Many situations have a good approximation \( \frac{d(V,y,Q)}{dx}=0 \)
  – Uniform flow
  – Look at extended Bernoulli equation
  – Friction slope exactly equal to bed slope

• Important characteristics
  – Flow area, \( A \)
  – Wetted Perimeter, \( P \)
  – Average Velocity, \( V \)
  – Wall Shear Stress, \( \tau_w \)
  – Bed Slope, \( S_0 \)
Apply standard force balance to section of channel
- End forces and velocities cancel
- Weight projection along channel direction balanced by shear stress along wetted perimeter
Shear Stress

• Like pipe flow, stress approximately proportional to square of velocity
  – Constant of proportionality dependent on bed roughness
  – Leads to Chezy coefficient
  – Has dimensions depending on units used, not dimensionless!

• Solution for velocity in uniform flow
Manning’s Equation

• Studies show that Chezy coefficient not exactly constant
  – Varies with hydraulic radius

• Leads to Manning’s equation
  – Also has dimensional units
  – Varies with type of bed (table)
  – Conversion factor between SI, BG units

• Manning’s equation more widely used
<table>
<thead>
<tr>
<th>Wetted Perimeter</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Natural channels</strong></td>
<td></td>
</tr>
<tr>
<td>Clean and straight</td>
<td>0.030</td>
</tr>
<tr>
<td>Sluggish with deep pools</td>
<td>0.040</td>
</tr>
<tr>
<td>Major rivers</td>
<td>0.035</td>
</tr>
<tr>
<td><strong>B. Floodplains</strong></td>
<td></td>
</tr>
<tr>
<td>Pasture, farmland</td>
<td>0.035</td>
</tr>
<tr>
<td>Light brush</td>
<td>0.050</td>
</tr>
<tr>
<td>Heavy brush</td>
<td>0.075</td>
</tr>
<tr>
<td>Trees</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>C. Excavated earth channels</strong></td>
<td></td>
</tr>
<tr>
<td>Clean</td>
<td>0.022</td>
</tr>
<tr>
<td>Gravelly</td>
<td>0.025</td>
</tr>
<tr>
<td>Weedy</td>
<td>0.030</td>
</tr>
<tr>
<td>Stony, cobbles</td>
<td>0.035</td>
</tr>
<tr>
<td><strong>D. Artificially lined channels</strong></td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td>0.010</td>
</tr>
<tr>
<td>Brass</td>
<td>0.011</td>
</tr>
<tr>
<td>Steel, smooth</td>
<td>0.012</td>
</tr>
<tr>
<td>Steel, painted</td>
<td>0.014</td>
</tr>
<tr>
<td>Steel, riveted</td>
<td>0.015</td>
</tr>
<tr>
<td>Cast iron</td>
<td>0.013</td>
</tr>
<tr>
<td>Concrete, finished</td>
<td>0.012</td>
</tr>
<tr>
<td>Concrete, unfinished</td>
<td>0.014</td>
</tr>
<tr>
<td>Planed wood</td>
<td>0.012</td>
</tr>
<tr>
<td>Clay tile</td>
<td>0.014</td>
</tr>
<tr>
<td>Brickwork</td>
<td>0.015</td>
</tr>
<tr>
<td>Asphalt</td>
<td>0.016</td>
</tr>
<tr>
<td>Corrugated metal</td>
<td>0.022</td>
</tr>
<tr>
<td>Rubble masonry</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Rivers have additional roughness-type dissipation from bends, bedforms, bars, etc.

High Manning’s $n$ for rough channels

Low Manning’s $n$ for smooth channels
Types of Channel Problems

1. Given slope, depth, channel characteristics
   - Find Flow Rate

2. Given flow rate, depth, channel characteristics
   - Find Design Slope

3. Given flow rate, slope, channel characteristics
   - Find Flow Depth
South Bend Last Week
Examples

• 10.2 Basic channel flow
• 10.4 Variable roughness – compound channel
• 10.5 Best hydraulic cross-section
  – (Minimum cross-sectional area for given flow rate)
Compound Channels

• Sometimes open channel flow may have two distinct parts
  – E.g. during flood have channel flow and overbank flow
  – Different roughness, Manning’s n

• To compute total flow, divide channel into sections
  – In each section, compute $A$, $P$, $R_h$, $V$, and $Q$
  – Add flows together to get total flow rate
Design for Unlined Channels
(Sediment Beds) Houghtalen 6.9.1
(handout)

• Concrete channels can resist any realistic bed stress
• Sediment channels will erode if the bed stress or bed velocity is too large
  – Maximum permissible velocity or maximum permissible stress
  – Varies with material
• Also maximum stable side slope
  – Varies with material
## Maximum Permissible Side Slopes

### Table 6.6 Houghtalen

<table>
<thead>
<tr>
<th>Material</th>
<th>Side Slope (H:V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>Nearly Vertical</td>
</tr>
<tr>
<td>Muck and Peat Soils</td>
<td>¼:1</td>
</tr>
<tr>
<td>Stiff clay, earth with concrete lining</td>
<td>½:1 to 1:1</td>
</tr>
<tr>
<td>Earth with stone lining or earth for large channels</td>
<td>1:1</td>
</tr>
<tr>
<td>Firm clay or earth for small ditches</td>
<td>1.5:1</td>
</tr>
<tr>
<td>Loose, sandy earth</td>
<td>2:1 to 4:1</td>
</tr>
<tr>
<td>Sandy loam or porous clay</td>
<td>3:1</td>
</tr>
</tbody>
</table>
Maximum Permissible Velocities
Table 6.7 Houghtalen

<table>
<thead>
<tr>
<th>Channel Material</th>
<th>$V_{\text{max}}$ (ft/s)</th>
<th>$V_{\text{max}}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand and Gravel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine Sand</td>
<td>2.0</td>
<td>0.6</td>
</tr>
<tr>
<td>Coarse Sand</td>
<td>4.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Fine Gravel</td>
<td>6.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Earth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sandy Silt</td>
<td>2.0</td>
<td>0.6</td>
</tr>
<tr>
<td>Silt Clay</td>
<td>3.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Clay</td>
<td>6.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>
Freeboard

- Add extra depth to channel above water surface to account for waves, irregularities, other unknowns

\[ F = \sqrt{C_y} \]

- C from 1.5 (20cfs) to 2.5 (3000cfs) US units
- C from 0.5 (0.6m³/s) to 0.76 (85m³/s) metric
- Use linear variation between these limits!
Design Procedure for Unlined Channels

1. Determine Manning’s $n$, maximum side slope, maximum velocity
2. Solve for hydraulic radius, $R_h$
3. Compute required flow area, $Q=AV$
4. Compute wetted perimeter $P=A/R_h$
5. Solve for flow depth, $y$, and bottom width, $b$, simultaneously
6. Check Froude number to make sure it is not close to 1
7. Add freeboard and modify for other criteria
Example

• Design channel for following conditions
  – Q = 9.0 m$^3$/s
  – Stiff clay
  – $S_0=0.0028$
Design for Lined Channels

- Concrete, grouted riprap, etc.
- No erodibility constraints for realistic channel velocities
- Use best hydraulic cross-section for trapezoidal section

\[
\frac{b}{y} = 2\left(\sqrt{1+m^2} - m\right)
\]
Design Procedure for Lined Channels

1. Select side slope, m and determine Manning’s n for bed material
2. Evaluate b/y for best cross-section
3. Solve rearranged Manning’s formula
   \[ y = \frac{\left(\frac{b}{y} + 2\sqrt{1 + m^2}\right)^{1/4}}{\left(\frac{b}{y} + m\right)^{5/8}} \left(\frac{Q_n}{\kappa \sqrt{S_0}}\right)^{3/8} \]
4. Check Froude Number
5. Determine Height of Lining from Diagram
Lined Channel Example

• Trapezoidal, concrete-lined channel (n=0.013) with Q=15m³/s, S₀=0.00095, side slope m=2.0 (2H:1V slope)
Gradually Varying Flow

- Often, flow depths, slopes are not constant
  - Steeper, gentler slope
  - Control point
- But time variations are usually quite slow over short distances
- Extend uniform channel solutions for slowly varying flow (in space)
- Also will allow us to determine how changes occur from one state to another (e.g. supercritical to subcritical)
Gradually Varied Flow Analysis

- Use Extended Bernoulli equation for channel flow, but not in complete equilibrium
- Rewrite in terms of friction slope, bed slope and Froude number
- Numerous other ways to write gradually varied flow equations
Increase in Energy, Constant Cross-Section

- Depth Increases for subcritical increasing E
- Depth Decreases for supercritical increasing E
Increase in Energy, Constant Cross-Section

- Depth Increases for subcritical increasing E
- Depth Increases for supercritical increasing E
Increase in E, Downstream Contraction

- Depth still Increases for subcritical increasing E
Increase in $E$, Downstream Contraction

- Depth might increase or decrease for supercritical increasing $E$
Important Concepts

• Normal depth $y_n$
  – Depth at which $S_0 = S_f$
  – Often do not need to know exactly, just if supercritical or subcritical

• Actual depth $y$

• Critical depth $y_c$
  – Depth at which $Fr = 1$ (may have to take into account non-rectangular channels)

• Relative depths of $y$, $y_n$, $y_c$ important for distinguishing the type of flow
Gradually Varying Capsule Summary

• Flow always tries to approach normal depth (natural flow depth)
  – Sometimes critical depth gets in the way first
• Subcritical flow approaches $y_n$ (or $y_c$) heading upstream
• Supercritical flow approaches $y_n$ (or $y_c$) heading downstream
• Which depth is approached depends on equilibrium conditions
  – Will try to approach equilibrium in the appropriate direction
• Horizontal and adverse slopes always approach critical point going downstream
Unsteady Flow Regions

- Region 1: Actual depth $y$ is greater than both critical depth $y_c$ and normal depth $y_n$
- Region 2: Actual depth $y$ is in between $y_c$ and $y_n$
- Region 3: Actual depth $y$ is less than both $y_c$ and $y_n$
Gradually Varied Flow Characteristics

• Two major categories
  – Steep vs Mild slopes (also Critical, Horizontal, Adverse)
  – Subcritical, Critical, Supercritical flow
  – Combination gives the overall picture
  – Different behaviors

• Critical points are of overwhelming importance

• Directions of influence
  – Supercritical – travels only downstream
  – Subcritical – both upstream and downstream
<table>
<thead>
<tr>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild slope $S &lt; S_c$</td>
<td>Mild slope $S &lt; S_c$</td>
<td>Mild Slope Region 3 (M3)</td>
</tr>
<tr>
<td>$y &lt; y_c$</td>
<td>$y &lt; y_c$</td>
<td>$y &lt; y_c$</td>
</tr>
<tr>
<td>Approaches $y_n$ downstream</td>
<td>Approaches $y_n$ downstream</td>
<td>Approaches $y_n$ downstream</td>
</tr>
<tr>
<td>Steep slope $S &gt; S_c$</td>
<td>Steep slope $S &gt; S_c$</td>
<td>Steep Slope Region 2 (S2)</td>
</tr>
<tr>
<td>$y_n &lt; y &lt; y_c$</td>
<td>$y_n &lt; y &lt; y_c$</td>
<td>$y_n &lt; y &lt; y_c$</td>
</tr>
<tr>
<td>Approaches $y_n$ downstream</td>
<td>Approaches $y_n$ downstream</td>
<td>Approaches $y_n$ downstream</td>
</tr>
<tr>
<td>Critical slope $S = S_c$</td>
<td>Critical slope $S = S_c$</td>
<td>Horizontal Slope Region 3 (H2)</td>
</tr>
<tr>
<td>$y = y_c$</td>
<td>$y = y_c$</td>
<td>$y &gt; y_c$</td>
</tr>
<tr>
<td>$y_n = y_c$</td>
<td>$y_n = y_c$</td>
<td>$y &lt; y_n (y_\infty)$</td>
</tr>
<tr>
<td>Approaches $y_n$ upstream</td>
<td>Approaches $y_n$ upstream</td>
<td>Approaches $y_n$ upstream</td>
</tr>
<tr>
<td>Horizontal slope $S = 0$</td>
<td>Horizontal slope $S = 0$</td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>None</td>
<td></td>
</tr>
<tr>
<td>Adverse slope</td>
<td>Adverse slope</td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>None</td>
<td></td>
</tr>
</tbody>
</table>
Flow Profiles

• Mild Slope
  – Subcritical flow approaches asymptote heading upstream (M1, M2 profiles)
  – Supercritical flow gets deeper heading downstream, unsustainable (M3)

• Steep Slope
  – Subcritical flow gets shallower heading upstream, unsustainable (S1)
  – Supercritical flow approaches asymptote heading downstream (S2, S3)
Numerical Integration

• Solve differential equation for y
  – Uses standard numerical techniques
  – Shows clearly effects of forcing, Froude number
    • Non-rectangular Froude number definition
  – Only (as we do it) for constant channel cross-section (prismatic channel)
Standard Step Method

• Choose $\Delta x$, solve for $\Delta E$, then $\Delta y$
  – Stay on the same branch of solution
  – May require iteration
  – Easy to account for changing channel shape

• Straightforward but may require solution of cubic equation to solve for $y$
  – Subcritical flow stable integration is upstream
  – Supercritical flow, iterate downstream
Direct Step Method

• Choose $\Delta y$, solve for $\Delta E$, then $\Delta x$
  – Careful to choose correct sign of $\Delta y$ to step in correct direction
  – No iteration required
  – If $(S_0-S_f)$ is small, $\Delta x$ may be too large
  – Subcritical flow stable integration is upstream
  – Supercritical flow, iterate downstream
Control Points

• Places where flow changes from subcritical to supercritical
  – Weir
  – Change in Slope
  – Spillways, etc

• Compute downstream from supercritical control, upstream from subcritical control

• Hydraulic jumps occur when upstream and downstream propagating characteristics reach their matching depths at the same point
Control Point Examples
What Flow Profiles are Possible?

• Assume subcritical flow at point A
Open Channel Summary

1. Rapidly Varying Flow
   - Specific Energy $E$ and Specific Flow $q$
   - Weirs
   - Hydraulic Jumps

2. Uniform Channel Flow
   - Gravity and Frictional Balance
   - Manning’s Equation: Area, Perimeter, Hydraulic Radius
   - Compound Channels
   - Channel Design
Summary (cont.)

3. Gradually Varying Flow
   - Specific Energy
   - Friction – Manning’s equation
   - Numerical Solution Methods
   - General flow profiles from flow characteristics
Course Topics

1. Pumps, Turbines and Pipe networks
   – Moment of Momentum Revisited
   – Types of pumps and turbines and uses
   – Pump-pipe systems, networks
2. Open Channel Flow
   – Specific Energy and Rapid Transitions
   – Hydraulic Jumps
   – Slowly varying flow
   – Open channel control structures
3. Introduction to Surface Water Hydrology
   – Hydrologic Cycle
   – Rainfall, Runoff and Design Events
4. Hydraulic Structures
   – Dams, weirs, spillways
   – Safety and Effects of Hydraulic Structures