The Realm of the Sacred, wherein We may not Draw an Inference from Something which Itself has been Inferred:

a reading of Talmud Bavli *Zevachim* folio 50 by

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The exegesis of sacred rites in the Talmud is subject to a restriction on the iteration and composition of inference rules. In order to determine the scope and limits of that restriction, the sages of the Talmud deploy those very same inference rules. We present the remarkable features of this early use of self-reference to navigate logical constraints and uncover the hidden complexity behind the sages' arguments. An Appendix contains a translation of the relevant *sugya*.

1. Introduction

Two favorite pursuits of modern logical research are the intricacies engendered by self-reference and the somewhat ironic phenomenon that unexpected complexity often results when one imposes constraints on one's methods. Logicians inspired by Epimenides to take up the former pursuit boast a panoply of creations ranging from the theorems of Tarski, Gödel, Löb, and Cantor to Russell's calamitous discovery about Frege's system. Socratic ironists have meanwhile drawn attention to the expressive strength of sub-structural logics and the computational features of bounded arithmetic. Particularly rewarding from an interpretive point of view have been the occasional exchanges between Athens and Crete: witness procedural (e.g., intuitionistic, linear) logics whose every theorem makes reference to its logic's own rules.

The Babylonian Talmud records a peculiar entanglement of self-reference and inferential restriction with no Greek pedigree. The text is the *sugya*[1] from tractate *Zevachim* demarcated "*hekeish*, *gezeirah shavah*, *kal vachomer*" concerning "double *limudim* in *kodshim*"—i.e., the possibility of iterating an inference rule or composing one inference rule with another when reasoning about sanctified objects or sacred rites. A reading of this *sugya* reveals a creative and subtle use of circularity, astonishing logical foresight, and novel ideas about how restricting methods might reveal logical complexity worthy of attention. Far from suggesting that the sages of the Talmud anticipated any modern logical preoccupation, the reading discloses the utter strangeness, from the modern point of view, of a once thriving inferential practice.

[1] A *sugya* is the basic unit of talmudic dialectical exchange. Compare a Platonic *dialog* or a Zen *koan*.

2. Background to the *sugya*

The inferential rules that the sages deploy throughout the Talmud are *hermeneutical* devices. Together they form an interpretive framework for extracting "*halachah*" (= the Way) from a fixed text, the Torah. The rules demand that the Torah be "read" often in extreme variance from, even disregard of, its apparent meaning in order to uncover the details of various injunctions, rites, obligations, etc. to which one who would follow the Way must adhere. As devices of hermeneutics principally intended to generate forensic data, these inferential principles are to be distinguished from rules of deductive consequence meant to preserve truth under modal variance. Indeed, together with the textual matter from which the Talmudic sages infer the nature of the Way, there exists an oral Torah of specific rites, obligations, etc. to which the sage ascribes the same divine origin as the written Torah, and the inferential principles themselves together with the details of how and when they are to be applied are part of this oral Torah—as opposed to, say, specifications of ways to get at the intended meaning of the Torah that strike one as reasonable. In other words, these principles need not "make any logical sense"—they demand the sage's mastery despite their tendency to defy rationalization for the single reason that the Torah itself has put them forward as the only reliable way to extract the details of the Way from the Torah.

Something circular is in the air, then, as soon as one encounters Talmudic inference, and the sages rest comfortably with it. At even its most self-reflective, Talmudic discussion never exhibits the sort of foundational aspirations that have characterized so much post-Hellenistic thought. Although this is not the circularity that the present study addresses, a basic awareness of it will surely enrich one's reading of the *sugya*. The most cursory familiarity of the inference rules themselves will doubtless also be of value.

To that end these few words should suffice: With *hekeish* and *gezeirah shavah* one uses textual cues to import known laws from one context to another. In using *hekeish*, typically, one observes that two terms or ideas are textually juxtaposed (or explicitly compared some other wise) with no obvious rationale—that is, a straight reading of the passage doesn't extract any legal nuance from the fact that this juxtaposition exists, and further, one can envision the passage being reworded more concisely, the juxtaposition done away with, without any apparent loss of meaning. An instance of *hekeish* under these circumstances allows one to infer some yet unknown law about one of the juxtaposed terms from the fact that one knows it to be true of the other term. Similar considerations govern *gezeirah shavah*. It differs in that one notices seemingly needless recurrence of words in two separate passages, upon which one infers some yet to be known law about the topic of one passage from the fact that it is already known about the topic of the other passage. A third inference rule not mentioned in the mnemonic of the *sugya* is *binyan av*, which operates under the assumption that some topic treated in detail in the Torah plays the role of a paradigm in the following sense: its legal details apply also to other topics that share its definitive characteristics but about which the Torah says comparably little. One may observe in the first half of the *sugya* (see

Appendix) specific instances of these rules.

Special attention must be paid to a fourth and final inference rule, *kal vachomer*. This is the rule that the sages repeatedly deploy in the *sugya* in attempted demonstrations that the restriction against iterated and composed inferences may be lifted. It is also the principle used explicitly to draw inferences about its own scope. As these two phenomena—how they arise and how the sages react to them—are the focus of this study, one must have a basic understanding of the rule itself. The expression "*kal vachomer*" translates as "weak and strong." The principle is used to answer in the affirmative (resp. negative) questions about whether a certain legal stringency (resp. leniency) applies in a given situation, as follows: (1) First one argues that the situation under consideration (the more "*chomer*" one) is stronger than another situation (i.e., that it is comparably "*kal*") in the sense that all the conceivably relevant details of the latter are present in the former. (2) Then one notes that stringency (resp. leniency) applies (or doesn't) in the *kal* situation. This supports the conclusion that it applies (resp. fails) in the *chomer* situation.

Two constraints govern every application of *kal vachomer*. First, the limits of conceivability in (1) above are nearly boundless: any forensically relevant difference ("*pirka*") between the alleged *kal* and *chomer* case can be cited to invalidate a *kal vachomer*. In other words, the *chomer* situation must be stronger in absolutely every way. Second, the inference itself is subject to the principle of "*daya*" which forbids one from inferring greater stringency into the *chomer* case than was noted in the *kal* case (this principle is invoked somewhat cryptically at 51b(3) by Rabbi Yosei.) Meanwhile, importantly, the sense in which one situation need exhibit "strength" over another in order for a *kal vachomer* argument to apply is quite liberal: there need be no obvious common-sense connection between the "strength" of the "*chomer*" situation and the legal detail that the argument would transfer.

The *sugya* "*hekeish*, *gezeirah shavah*, *kal vachomer*" deals with a single problem. Talmudic hermeneutics treats, as one would expect, all details of the Way inferred through the deployment of one of these four principles as settled law on equal footing with any explicit teaching in the oral or written Torah. For that reason, matters known only through inference are generally available to "return as premises" in future inferences—i.e., inferences viewed as functions can be iterated and composed with one another indefinitely. But this sensible feature of talmudic logic is suspended when reasoning about matters that pertain to sacrificial rites, sacred objects, and behavior in the Temple. "In the realm of the sacred," Rabbi Yochanan reports at 49b(-5), "we may not draw an inference from something which itself has been inferred." This pronouncement must not be understood quite literally, for the *sugya* establishes that several patterns of composition and iteration indeed are permitted when reasoning about sacred affairs. Rabbi Yochanan means simply that the blanket permissibility of composition and iteration does not apply when reasoning about sacred affairs, that one must instead seek independent justification before attempting to construct chains of inferences about such matters. In the absence of a general license to infer sequentially, the *sugya* steps through each possible combination of inferences, seeking to demonstrate (or report from a teaching recorded elsewhere) its validity or invalidity.

The very nature of the *sugya*'s problem will strike a modern reader as odd on at least two counts. One wonders why the laws governing logic should vary from domain to domain. The individual inference rules, with the possible exception of *kal vachomer*, don't exactly force themselves upon one as one is likely accustomed to expect from logical principles. Still, there seems to be a significant difference between being asked to embrace them as an appropriate way to come to understand the world and being asked to do so only some of the time. This worry may be dampened by reflecting again on the role of talmudic hermeneutics. These principles are not on offer as laws of deductive consequence, but rather as instruments for uncovering the details of the Way. The Hebrew word for sanctity is "*kedusha*," which carries strong suggestions of separateness and otherness, a connotation one can see reflected here in the fact that laws about the sacred are themselves "*kodesh*" so that the principles governing how even to learn them differ from principles of reasoning about the Way generally. Reasoning about sacred affairs is certainly made strange by the requirement that one not only keep track of what one knows but also, for each fact, of how one came to know it. This constraint has dramatic consequences on how the *sugya* unfolds and is to be singled out for that reason, but neither it nor the fact that it is only in place some of the time are fundamentally less sensible than the individual inference rules themselves.

Secondly, one might be surprised by the question of the iterability of *kal vachomer*. After all, this question seems initially to be answered by the nature of the rule itself. For the relation "being strictly more *chomer* than" surely is transitive. Thus, unlike the other rules, each of whose iterability is essential for reaching certain inferences, one should seemingly always be able to substitute for a chain of *kal vachomer*'s with initial premise ϕ and conclusion ψ a single *kal vachomer* from ϕ to ψ (think of "multi-cut"). The commentary *Birkat Hazevach* raises this question, which is taken up in section five of the present study.

3. Which compositions are sanctioned?

In order to represent the *sugya*'s arguments vividly, the following abbreviations will be used:

$$\begin{array}{rcl} \text{k} & = & \textit{kal vachomer} \\ \text{g} & = & \textit{gezeirah shavah} \\ \text{h} & = & \textit{hekeish} \\ \text{b} & = & \textit{binyan av} \end{array}$$

Further, the two-place relation symbol $\rightarrow$ will be used to express the fact that the function on the right composes with the function on the left—e.g., $h \rightarrow k$ means that something inferred with *hekeish* can return and serve as a premise of a *kal vachomer*. The denial of such a claim is expressed with the symbol $\nrightarrow$.

In basic outline, the *sugya* unfolds as follows. The discussion commences with two facts already established: $h \nrightarrow h$ and $h \to k$. Various sages quote earlier authoritative teachings as evidence that other compositions are or aren't possible. Several such textual proofs are challenged along the way, sometimes successfully and other times not. Eventually it is established that $h \nrightarrow g$ and $g \to g$. Rav Papa offers one teaching as an instance of $g \to h$, but the sages disagree about its pertinence. The disagreement amounts to this: Rav Papa cites (1) a use of *gezeirah shavah* from a law about tithed grain to a law about the *shelamim* sacrifice and (2) a use of *hekeish* to import that law from the context of the *shelamim* sacrifice to the context of the *todah* sacrifice. The sages agree about what these teachings involve, but they disagree about whether their composition is of the sort that Rabbi Yochanan's report would *prima facie* rule out. Though the *shelamim* and *todah* sacrifices have sanctity, tithed grain does not. For this reason Mar Zutra does not count Rav Papa's example as a case of composition of inferences about the sacred. Ravina, however, maintains that any inference whose *conclusion* is a law about the sacred realm qualifies as an inference in that realm. Based on this understanding, Ravina defends Rav Papa's example as an instance of $g \to h$ in the realm of the sacred.

At 50b(3) the discussion takes a dramatic turn when, to determine whether or not *kal vachomer* can pick up from a *gezeirah shavah*, rather than report any specific teaching as evidence one way or the other, the sages deploy the *kal vachomer* rule itself. The argument is that (1) *gezeirah shavah* is a stronger inference rule than *hekeish* because of the fact that $g \to h$ while $h \nrightarrow h$, and that (2) despite its relative weakness, $h \to k$. An application of *kal vachomer* then guarantees that $g \to k$.

The sages point out that this leaves $g \to k$ subject to the disagreement between Mar Zutra and Ravina, in so far as one of the inference's premises is the very thing which proved contentious before. More noteworthy than the sages' verdict, though, is their strategy. This use of *kal vachomer* to discover something about itself is unprecedented and inventive. For that reason encountering it can be jarring initially. But after the dust settles, the inference is attractive: the fact that one rule's consequences have a wide range of use as premises in further argumentation does make that rule appear to that extent strong. If one rule is known to be stronger than another in this sense, then that does seem like grounds to designate it as *chomer* and the other as *kal*. Underlying this reasoning pattern is the principle that *if the conclusions of one inference rule* (A) *can serve as premises of another inference rule* (B) *although the conclusions of a third inference rule* (C) *cannot so serve, then any inference rule that can take as premises the conclusions of* (C) *can also take as premises the conclusions of* (A). Label this PRINCIPLE 1.

The sages don't invoke PRINCIPLE 1 other than on this one occasion. More often they invoke the following PRINCIPLE 2: *if one inference rule* (A) *can take as premises the conclusions of another inference rule* (B) *although a third inference rule* (C) *cannot take the conclusions of* (B) *as premises, then any inference rule that can take as premises the conclusions of* (C) *can also take as premises the conclusions of* (A). This principle underlies the sages' demonstrations at 50b(16) and 50b(23), again driven by *kal vachomer* arguments, of $k \rightarrow h$ and $k \rightarrow g$, as well as their successful proof of $k \rightarrow k$ at 50b(-14). But because its attempted deployment at 50b(-19) to prove $k \rightarrow k$ is the focus of section five below, it is preferable to illustrate PRINCIPLE 2 with that argument despite the fact that the sages find it inconclusive: The argument is that (1) *kal vachomer* is a stronger inference principle than *gezeirah shavah* because of the fact that $h \rightarrow k$ while $h \nrightarrow g$, and that (2) despite its relative weakness, $g \rightarrow k$. An application of *kal vachomer* then guarantees that $k \rightarrow k$.

Despite the formal analogy between this argument and the arguments that illustrate PRINCIPLE 1, something unquestionably new is at play here. PRINCIPLE 1 argues from A's evident superior ability to C's to lead into other inference rules to the likelihood, or certainty, that A will lead into any inference rule that C is known to lead into. Because the "evidence" in that premise is but a single inference rule that A leads into but C does not, PRINCIPLE 1 is far from obvious independent of any other assumptions about reasoning in the realm of the sacred. It is nevertheless intuitively compelling. PRINCIPLE 2 is another matter, though, for it asks one to conclude from A's evident superior ability to C's to *pick up from* other inference rules that A will lead into any inference rule that C is known to lead into. While many objects, like magnets, might have strongly correlated capacities to give and receive, many others, such as blood donors, do not. Thus the sages' invocation of PRINCIPLE 2 seems to reveal some non-trivial, and not especially intuitive, received knowledge on their part of the nature of sacred reasoning.

This observation could foster a provocative interpretation of the *sugya*: Rabbi Yochanan reports that reasoning about sacred matters is subject to general constraints any exceptions to which must be individually defended. But that does not place the sages in complete ignorance about the the nature of the inference rules. Not knowing in advance which functions compose with which others does not leave one in the dark about all aspects of those functions' behavior. For example, one could still know that for all functions x and y, if $x \rightarrow y$ then $y \rightarrow x$. (In fact, the sages not only don't know this about sacred inference, they apparently know it to be false: Ravina is able to maintain that, although $h \nrightarrow g$, $g \rightarrow h$. Likewise, in arguing against this position, Mar Zutra is committed both to $h \rightarrow k$ and $k \nrightarrow h$.) The sages proceed by setting up *kal vachomer* arguments in order to learn which of their rules compose with which others, and in their doing so—implicitly through the details of how they design their arguments—they reveal to us other general properties of talmudic logic that apply when reasoning about sacred matters despite the constraints that they are navigating. Can this interpretation be defended?

One might be inclined to reject it in light of yet another dizzying deployment of *kal vachomer* to delineate the scope of sacred inference. Recall that the sages' argument for $g \rightarrow k$ at 50b(4) is deemed inconclusive. At 50b(10) they present the following argument in its place: (1) *gezeirah shavah* is a stronger inference principle than *hekeish* because of the fact that $g \rightarrow g$ while $h \nrightarrow h$. (2) Despite its relative weakness, $h \rightarrow k$. An application of *kal vachomer* then guarantees that $g \rightarrow k$.

At first glance it might not be evident what principle underlies this argument. Presumably to address that mystery, the sages include the extra word "*chavertah*" (= its fellow) here, thereby emphasizing that the evident strength of *gezeirah shavah* lies in its self-iterability. At work is PRINCIPLE 3: *if one inference rule* (A) *is self-iterable while a second inference rule* (B) *is not, then any inference rule that can take as premises the conclusions of* (B) *can also take as premises the conclusions of* (A).

What essential property of talmudic reasoning might the sages be teaching through this principle? Because it is not easy to imagine that any essential feature of inference is embodied in this principle, one might reasonably suspect that, far from proceeding strategically in order to disclose details of sacred reason, the sages are designing their arguments entirely *creatively*. This suspicion fosters quite a different interpretation of the *sugya*: Rabbi Yochanan's dictum was wholly devastating to any preconception of the behaviors of the rules of reasoning. The sages, thus blinded in the presence of the foreign ways of sacrament and ritual, latch onto whatever distinctions they can find in order to designate some principles *chomer*, others *kal* based on these distinctions. They thereby replenish their logical armory in the only way available.

But what licenses this creativity? This question is fueled by the observation that, unlike PRINCIPLES 1 and 2, each of which measures inferential strength against a fixed standard, PRINCIPLE 3 measures inferential strength against an *indexical* standard. If each rule's range of composition is determined by simultaneous reference to how it behaves with respect to itself and how it behaves with respect to the entire collective of rules, might one risk inconsistency? Does absolutely anything go in this arena? If so, consider the effect of labeling "strong" also (4) those rules that lead into more rules than they pick up from as well as (5) those that pick up from all self-iterable rules, and (6) those that lead into every rule that they pick up from, ... One need not be too inventive to find oneself very soon in the awkward predicament of having a categorical set of principles whose only model is the trivial one, familiar from mundane inferential practice—in this case Rabbi Yochanan's dictum no longer has any meaning. As explained in section two, a *kal vachomer* argument need not pivot on a "strength" of one scenario over another that is evidently or intuitively connected to the law being established about that scenario. But if a barrage of *kal vachomer* arguments about the same phenomena are to cite different properties as strengths *en route* to drawing similar conclusions, then those properties had better be consistent with one another for this argumentative strategy to be coherent. This question is addressed in section four.

The *sugya* ends with a string of unresolved queries about how the conclusions of *binyan av* inferences fill the role of premise in further argumentation. This complements the fact that throughout the discussion, questions about whether a principle's conclusions can be picked up by a *binyan av* have gone unanswered. Two partial suggestions of the dynamics of sacred reason have come to the fore:

According to Ravina

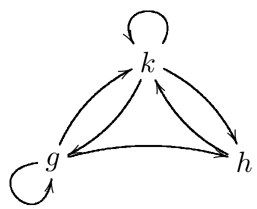
$h \nrightarrow h$	$h \nrightarrow g$	$h \rightarrow k$
$g \rightarrow h$	$g \rightarrow g$	$g \rightarrow k$
$k \rightarrow h$	$k \rightarrow g$	$k \rightarrow k$

According to Mar Zutra

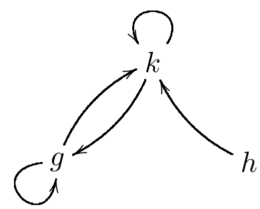
$h \nrightarrow h$	$h \nrightarrow g$	$h \rightarrow k$
$g \nrightarrow h$	$g \rightarrow g$	$g \rightarrow k$
$k \nrightarrow h$	$k \rightarrow g$	$k \rightarrow k$

These can be represented diagrammatically:

According to Ravina



According to Mar Zutra



4. How many principles?

Let us look more closely at the variety of principles the sages appeal to in constructing *kal vachomer* arguments about the behavior of their inference rules. Section three left us with a fundamental ambiguity about the sages' methodology. Can any distinction be appealed to in the designation of one rule as *kal*, another as *chomer*? This seems at times to be the case—especially with the appeal to PRINCIPLE 3 at 50b(10)—but this reading of the *sugya* was seen to be problematic: simultaneous recognition of multiple standards and self-reference are auspices of inconsistency better not to leave unchecked. On the other hand, if the sages' principles can be motivated by some systematic coherence, then a reading of the *sugya* emerges in which the sages offer a glimpse of the nature of sacred inference beyond mere facts of compositionality. Some attention to the principles the sages invoke and the order in which they do so should sort out this ambiguity in favor of the latter interpretation.

For the sake of clarity, we temporarily stop reading "$g \rightarrow h$" as "the conclusion of a *gezeirah shavah* can return as a premise of a *hekeish*" or even the more colloquial expressions from the previous section "*gezeirah shavah* can lead into *hekeish*" or "*hekeish* can pick up from *gezeirah shavah*." Instead we shall read "$g \rightarrow h$" as "g teaches h" or "h learns from g," and we shall personify inference principles by referring to them with agent pronouns and, occasionally, personal names[2].

First consider PRINCIPLE 1. Its formulation in quantification theory is,

1. $\forall x \forall y((x \rightarrow y) \supset \forall z(\exists w((z \rightarrow w) \wedge (x \nrightarrow w)) \supset (z \rightarrow y)))$

[2]Interestingly, the sages' own expression of this relation "*davar halameid b'hekeish mahu, shelameid b'gezeirah shavah*?" would be literally translated in just this way.

which might be read, “if x teaches y, then so too must everyone who teaches someone x doesn’t teach.” PRINCIPLE 1 thus expresses the fact persons, viewed as students, are linearly ordered according to teachability. Let us say, to illustrate this idea, that Reuven is fairly hard to teach—certainly harder than Shimon and Levi—but that I am known to teach him. One could infer, by PRINCIPLE 1, that I also teach Shimon and Levi.

Accepting this principle ushers in others at once. For example, it is not obvious that persons should be linearly ordered, given that they are in terms of teachability, also according to how hard it is to learn from them. But it is true.

Consider,

4 $\forall x \forall y((y \rightarrow x) \supset \forall z(\exists w((w \rightarrow z) \wedge (w \nrightarrow x)) \supset (y \rightarrow z)))$

This PRINCIPLE 4 says that if x learns from y, then so too must everyone who learns from someone from whom x doesn’t learn. One can check that it is logically equivalent to PRINCIPLE 1:

Theorem 1. PRINCIPLES *1 and 4 are first-order equivalences.*

Proof. To show that PRINCIPLE 4 follows from PRINCIPLE 1, suppose Reuven learns from Shimon. We must show that everyone who learns from someone Reuven doesn’t learn from must also learn from Shimon. Suppose that Levi learns from Yehuda, but that Reuven doesn’t. Then Shimon teaches Reuven, and Yehuda teaches Levi but not Reuven. By PRINCIPLE 1, it follows that Shimon teaches Levi. (Arguing in reverse is straightforward.) □

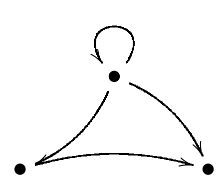
How does PRINCIPLE 2 relate to these? For clarity’s sake, let us also formulate this principle,

2 $\forall x \forall y((x \rightarrow y) \supset \forall z(\exists w((w \rightarrow z) \wedge (w \nrightarrow x)) \supset (z \rightarrow y)))$

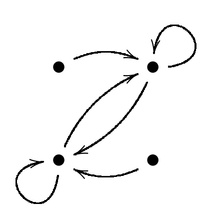
and observe its informal reading, “If x teaches y, then so too must everyone who learns from someone from whom x doesn’t learn.” This is harder to think about geometrically. Rather than try to characterize its models, though, it is more helpful simply to note how it constrains the models of PRINCIPLE 1 (and PRINCIPLE 4.)

First observe that PRINCIPLES 1 and 2 are logically independent:

a model of 1 in which 2 fails



a model of 2 in which 1 fails



Now observe that a fifth principle,

5 $\forall x \forall y((y \to x) \supset \forall z(\exists w((z \to w) \wedge (x \nrightarrow w)) \supset (y \to z)))$

which says that if x learns from y, then so too must everyone who teaches someone x doesn't teach, is logically equivalent to PRINCIPLE 2:

Theorem 2. PRINCIPLES *2 and 5 are first-order equivalences.*

Proof. To show that PRINCIPLE 5 follows from PRINCIPLE 2, suppose Reuven learns from Shimon. We must show that everyone who teaches someone Reuven doesn't teach must also learn from Shimon. Suppose Levi teaches Yehuda, but Reuven doesn't. Suppose now that Levi does not learn from Shimon, though he teaches Yehuda. Then by PRINCIPLE 2, Reuven, who does learn from Shimon, teaches Yehuda. Because this contradicts our first assumption, it follows that Levi does learn from Shimon. (Arguing in the reverse is straightforward.) □

We see that according to PRINCIPLES 2 and 5 persons, viewed again as students, are linearly ordered according to the amount of tutelage one needs in order to teach them. Thus, if Reuven is fairly hard to teach—evident from the fact that neither Shimon nor Levi manage to teach him—then in order ever to teach him, I need to learn from everyone Shimon and Levi have learned from. And we see that (equivalently) persons are linearly ordered by the students one needs to teach in order ever to learn from them.

Thus by endorsing both PRINCIPLES 1 and 2, one superimposes the two linear orderings with which we began, "ease to teach" and "ease to learn from." There are many ways to do this, but there are two over-arching constraints that characterize all models of these two principles. To express these constraints, define for each element a in the model (person) two sets of elements of the model, $S_a = \{x : a \to x\}$ (students of a) and $T_a = \{x : x \to a\}$ (teachers of a). (1) For all a, b, $S_a \subseteq S_b$ or $S_b \subseteq S_a$ and $T_a \subseteq T_b$ or $T_b \subseteq T_a$. (2) $S_a \subseteq S_b$ if, and only if, $T_a \subseteq T_b$. In other words, any two persons must be unambiguously comparable in terms of their degree of education and in terms of their success at teaching, and it cannot happen that one of them is superior in the first comparison but inferior in the second.

This is a bit of a mouth-ful, to be sure, but the point is that dwelling on it for a while does leave a fairly vivid impression. Just imagine a world where everyone can truthfully say "I'm a better teacher than you if, but only if, I'm better informed." That is a weaker push-pull correlation than one finds in magnets, because it quite reasonably leaves open the possibility of the occasional student from whom one learns nothing. It is far from empty, though, especially in its linearity. Most importantly, it is manifestly consistent.

It would seem, then, that the sages have devised their *kal vachomer* arguments in response, not to whimsical and inventive distinctions wherever they can find them, but to a clear view of something essential about the principles of talmudic hermeneutics: a robust standard of strength. PRINCIPLES 1 and 2 are hewn from this single idea, which, if true, makes those principles unassailable standards for *kal vachomer* arguments. This is the impression that we briefly considered in section three, before encountering the indexical standard of PRINCIPLE 3:

$$3 \;\; \forall x((x \to x) \supset \forall y(\exists z((z \nrightarrow z) \wedge (z \to y)) \supset (x \to y))).$$

It is remarkable, at first glance, that PRINCIPLE 3 would be invoked at all in the proof of $g \to k$. Recall that the sages' first attempt at this is based on an appeal to PRINCIPLE 1. This is deemed inconclusive because it depends on Ravina's contested interpretation of Rabbi Yochanan's dictum. What they did not do in the wake of this refutation was appeal again to PRINCIPLE 1 (*via* PRINCIPLE 4), as follows: (1) *kal vachomer* is a stronger inference principle than *gezeirah shavah* because of the fact that $h \to k$ while $h \nrightarrow g$, and (2) despite its relative weakness, $g \to g$. An application of *kal vachomer* then guarantees that $g \to k$. Because the sages pass over that argument despite its availability, in favor of an argument not at all evidently related to the concept that might have motivated PRINCIPLES 1 and 2, the image of the sages spinning *kal vachomer* arguments out of anything they can find recurs. And with this image re-emerges the impression that some safeguard against inconsistency is called for.

But this is a mirage. The sages' turn to PRINCIPLE 3 is neither an abandonment of the single idea that gives rise to the earlier principles nor a sign that they proceed unencumbered by any idea. For PRINCIPLE 3 is in fact a logical consequence of PRINCIPLE 2, as we now show.

Theorem 3. PRINCIPLE *3 is a first-order consequence of* PRINCIPLE *2.*

Proof. Consider the minimal diagram satisfying $a \to a$, $b \nrightarrow b$, and $b \to c$:

$$\begin{tikzcd}
 & a \arrow[loop above] & \\
b \arrow[rr, bend left=10] & & c
\end{tikzcd}$$

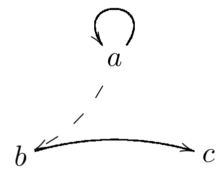
We will show that $a \to c$.

By PRINCIPLE 2 there are two problems.

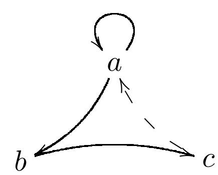
1. a learns from someone b doesn't learn from and yet b teaches someone a doesn't teach;

2. c learns from someone a doesn't learn from and yet a teaches someone c doesn't teach.

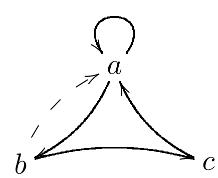
To address the first problem, either have $a \to c$ or $a \to b$. If $a \to c$, then we're done. Otherwise we have:



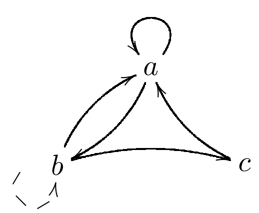
and we still must address the second problem. To do so, either have $c \to a$ or $b \to a$. In the first case we have,



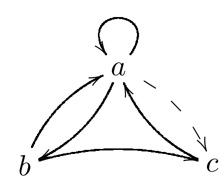
and a new problem emerges: b learns from someone from whom c doesn't learn, but c teaches someone b doesn't teach. To address this, either have $a \to c$ (and be done) or have $b \to a$, yielding,



This diagram now fails to satisfy PRINCIPLE 5 (hence also PRINCIPLE 2), for b teaches someone a doesn't teach but a learns from someone b doesn't learn from. We must have one of two possibilities:

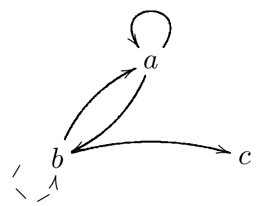


contradicting a hypothesis

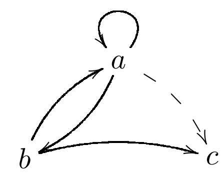


satisfying the goal $a \to c$

These last diagrams represent also the situation encountered if, in response to the second of the original problems, one has $b \to a$ instead of $c \to a$:



contradicting a hypothesis



satisfying the goal $a \to c$

$\square$

The sages' use of *kal vachomer* to prove facts about itself and its fellow inference rules appears almost chaotic on the surface. A closer look has revealed that the fanciful loops and textual creativity of talmudic self-reference are the outgrowth of a hidden methodological sobriety. A single idea—that the inference rules are linearly ordered by the rules they compose with on the right and on the left—motivates all the *kal vachomer* arguments in the *sugya*.

5. A son of a son of a *kal vachomer*

The high-water mark of the *sugya* is the refutation of the sages' first attempt to prove $k \to k$. The refutation is brief but definite. After presenting their initial argument, the sages remark at 50b(-16), "And this is a *kal vachomer*, a son of a *kal vachomer*." Immediately, they add, "It is a son of a son of a *kal vachomer*!" What exactly is the complaint being voiced here?

All commentators read these two lines, in one way or another, as a refutation of the argument they follow (the argument was presented in section three). They offer two fundamentally different explanations of what the error of this argument is, though, and of what the sages' words express. Here we look at these readings as presented by the two most authoritative commentators, Rabbi Shlomo ben Yitzchak ("Rashi") and the Tosafot.

Rashi explains that the initial remark is a comment about the (soon to be rejected) proof itself. We saw in section three that this was a proof by *kal vachomer*, establishing $k \to k$ from the fact that $g \to k$ despite the evident weakness of g relative to k. But we also saw that the key premise of this proof—the fact that $g \to k$—was itself established by *kal vachomer*. Indeed, this was the proof based on PRINCIPLE 3. Its key premise was the fact that $h \to k$ despite the evident weakness of h relative to g. Thus we have an inference chain

$$\cfrac{\cfrac{h \to k}{g \to k}\,k_2}{k \to k}\,k_1$$

wherein $g \to k$ serves as the conclusion of one *kal vachomer* (k_2) and as the key premise in another (k_1). Thus the inference k_1 is "a *kal vachomer*, a son of a *kal vachomer*," and, according to Rashi, the sages are pointing this out.

Notice, however, what happens if one applies what is learned in this proof and, in the course of reasoning about sacred matters, uses a conclusion of a *kal vachomer* as a premise of another *kal vachomer*. Suppose some law about the *chatat* sacrifice has been established with a *kal vachomer* from the case of the *shelamim* sacrifice. One wants now to establish that same law about the Day of Atonement sacrifice. Superficially, the chain of inferences has the structure:

$$\cfrac{\cfrac{\text{shelamim}}{\text{chatat}}\,k_2}{\text{atonement}}\,k_1$$

This seems to be just the sort of inference sanctioned by the sages' argument. But actually, there is a hidden premise of k_1, namely, the very fact that $k_2 \to k_1$, which is what the sages' proof allows. Thus, in full detail, the chain of inferences has the structure:

$$\cfrac{\cfrac{\text{shelamim}}{\text{chatat}}\,k_2 \qquad \cfrac{\cfrac{h \to k}{g \to k}\,k_4}{k_2 \to k_1}\,k_3}{\text{atonement}}\,k_1$$

If with all hidden structure displayed one looks again at k_1, one observes that it is the "son" not only of k_2 but also of k_3. Since k_3 is, meanwhile, the "son" of k_4, one needs to know, in order to iterate *kal vachomer* arguments about sacred matters, not only that one reiteration but that two successive reiterations are admissible. This, though, the sages never argued for. By making any attempted iteration of the *kal vachomer* rule actually a chain of two successive reiterations of that rule, the sages failed to prove even that one reiteration is admissible. This, according to Rashi, is the objection voiced in the sages' comment "a son of a son of a *kal vachomer*!"

Rashi's idea that $k_2 \to k_1$ is a hidden premise of the inference k_1 is astonishing. Modern logicians will be familiar with the consequences of this idea from Lewis Carroll's celebrated essay "What the tortoise said to Achilles." But when the Tosafot take exception to Rashi's reading of the *sugya*, this is not the feature that catches their attention. We will soon see, in fact, that they agree with Rashi on this point. Instead they object that they don't see why, once a rule has been shown to be self-iterable, any sequence of iterations, however long, shouldn't be possible.

How, then, do the Tosafot suggest understanding the sages' cryptic remark? Simply from the evident circularity in the sages' argument:

$$\cfrac{\cfrac{h \to k}{g \to k}\,k_2}{k \to k}\,k_1$$

For there $k \to k$ is established by an explicit appeal to that very fact, with the use of the conclusion of k_2 as a premise of k_1. The Tosafot claim that k_1 itself rests on the (hidden) assumption $k_2 \to k_1$ (as the tortoise insisted to Achilles), but that this assumption is unwarranted, being an instance of k_1's own conclusion. Thus k_1, which the sages first point out to be "a *kal vachomer*, a son of a *kal vachomer*," is itself "a *kal vachomer*, a son of a son of a *kal vachomer*" (moreover, though the Tosafot don't say this outright, k_1 is its own grandfather.)[3]

One may justifiably wonder why Rabbi Yochanan's dictum should apply to an inference like k_1 in the above argument. For although it is manifestly an attempt to use the conclusion of a *kal vachomer* as (one of) its own premise(s), it is not obviously an attempt to do so in the course of reasoning about sacred matters. k_1, like k_2, is an inference about inference rules, not about sacrament. Shouldn't the blanket license on unrestricted self-iteration and composition apply in the case of such "meta-inferences?"

It might appear initially that the best answer to this question favors Ravina's understanding of Rabbi Yochanan's dictum. Both Rashi and the Tosafot have explained that an iterated *kal vachomer* argument about sacred matters actually rests on a hidden premise, itself a conclusion of an iterated *kal vachomer* argument. But even if, for example in the argument,

$$\cfrac{\cfrac{\text{shelamim}}{\text{chatat}}\,k_2 \qquad \cfrac{\cfrac{h \to k}{g \to k}\,k_4}{k_2 \to k_1}\,k_3}{\text{atonement}}\,k_1$$

the inferences k_3 and k_4 are not accounted as inferences about sacred affairs, perhaps because of their status as meta-inferences, the chain k_4, k_3, k_1 is subject to Rabbi Yochanan's dictum, at least according to Ravina, by virtue of the sacred topic of its conclusion ("atonement"). This observation applies more obviously to Rashi's reading of the *sugya*. It may also apply to the Tosafot's reading. For even if in the above argument k_4 and k_3 aren't subject to any restrictions up front, any attempt to apply their joint conclusion ($k \to k$) as a hidden premise in an inference about sacred matters induces such restrictions, according to Ravina, and thereby invalidates their composition for the reasons of circularity that the Tosafot point out.

[3]On the surface, it appears that Rashi's explanation of the passage is truer to its words, for he explicitly points out the "third generation" status of an inference while the Tosafot emphasize what seems to be a different point, *viz.* circularity. Not only has the present analysis shown how the Tosafot's reading fits with the Talmud's language, it has shown it to be in one sense a better fit: on their reading, the same inference (k_1) that the sages first call "a son of a *kal vachomer*" is the inference that they call "a son of a son of a *kal vachomer*." On Rashi's reading, we saw, the reference of these two remarks shifts.

This answer is problematic primarily because neither Rashi nor the Tosafot—nor, for that matter, the sages in the Talmud—suggest that the "son of a son of a *kal vachomer*" objection is based on Ravina's understanding. It is furthermore unclear that on Ravina's understanding an inference should become subject to the restrictions on reasoning about sacred matters because of how it is embedded in larger argumentative contexts. It is possible, perhaps more likely, that Ravina understands individual inferences as being subject to, or free from, those restrictions on their own merits, independent of such considerations.

A better answer seems to be simply that the subject matter of k_1 and k_2 is not talmudic logic *tout court*, but what we have occasionally designated "sacred inference." For the same reason that restrictions apply in the first place to sacred inference, they apply to any reasoning *about* sacred inference: reasoning about the Way in sacred matters has its own sanctity. Much modern logical research is driven by the idea that one's meta-logic be subject to fewer (Tarski) or more (Hilbert) restrictions than the logical systems under invesitgation. By contrast, in keeping with their willingness to treat the conclusions of "meta-inferences" as premises of inferences at the "object" level, the sages take it for granted that the logic behind one's reasoning about logical rules should be the same logic as what one is reasoning about.

Consider, finally, the question of the *Birkat Hazevach*: why are the sages preoccupied with establishing the self-iterability of *kal vachomer* in the first place, given its evident transitivity? The late commentators have suggested erudite and subtle answers to this question that would strike most modern readers as deeply scholastic. The present analysis uncovers a comparably straightforward answer. Both Rashi and the Tosafot point out that a instance of *kal vachomer* may have certain hidden premises, which could themselves be conclusions of *kal vachomer* inferences. But the content of such premises, we have seen, is of an entirely other nature than what is inferred from them. Consider such an expanded argument one final time:

$$\frac{\dfrac{\text{shelamim}}{\text{chatat}}\,k_2 \qquad \dfrac{\dfrac{h \to k}{g \to k}\,k_4}{k_2 \to k_1}\,k_3}{\text{atonement}}\,k_1$$

If *kal vachomer*, as it seems to be, is transitive, then this entire argument could be reduced to

$$\frac{\text{shelamim}}{\text{atonement}}\,k$$

However, even granting the transitivity of *kal vachomer*, the following expanded argument with only slightly different structure could not be reduced,

$$\frac{\dfrac{\text{shelamim}}{\text{chatat}}\,g \qquad \dfrac{h \to k}{g \to k_1}\,k_2}{\text{atonement}}\,k_1$$

The reason for the irreducibility of the chain k_2, k_1 is clear: transitivity fails along an inference's "hidden ancestry." Could this in fact be the intent behind the sages' announcement at 50b(-16)? "By the way, you might be wondering what a questionable case of iterated *kal vachomer* might be, given its evident transitivity. Observe that 'this' very use of *kal vachomer* to reason about itself, because of its hidden premises, 'is' an example of 'a *kal vachomer* a son of a *kal vachomer*,' of the sort that we are asking about: because it cannot be reduced, it needs justification!"

6. Other circularities?

The "hidden premise" reading put forward by Rashi and the Tosafot clarifies a potentially disorienting exchange in the *sugya* and opens up the sages' understanding of self-reference to rewarding insights. But one may worry that pressing this reading reveals more problematic circularities elsewhere in the *sugya*. The erroneous argument at 50b(-19) is not the only one with hidden premises. The earlier arguments for $g \to k$, $k \to h$, and $k \to g$ have the same structure.

In order to appreciate this concern, consider what would happen had the sages established $h \to k$ with a *kal vachomer* argument. Their proof would have had the basic structure:

$$\frac{\text{some premise}}{h \to k}\,k$$

But in its expanded form it would appear like this:

$$\frac{\dfrac{\text{some premise}}{\text{a first inference}}\,h \qquad \dfrac{\text{some premise}}{h \to k}\,k}{\text{a second inference}}\,k$$

Thus in order ever to apply *kal vachomer* to the conclusion of a *hekeish*, one would have to know, not only that this composition of inferences is permissible, but also that the self-iteration of *kal vachomer* is. The sages haven't established that second fact at the time that they address the question of $h \to k$ (for this question is regarded as settled at the onset of the *sugya*), so the proper reading of our imagined proof of $h \to k$ would have to be that the composition is permissible provided that $k \to k$ also is.

As the *sugya* unfolds, the sages do eventually establish that $k \to k$. At that moment, the proviso on our imaged proof of $h \to k$ could seemingly be dropped. However, that would be a mistake. The sages' proof of $k \to k$ has as its explicit premise the previously established fact $h \to k$. But in our imagined scenario, that fact hasn't been established and in fact depends on $k \to k$—the very fact that the sages infer from it. This is the sort of circularity to which the hidden premises reading might be susceptible.

It turns out that the sages' arguments do exhibit this sort of circularity at one point. For their proof of $g \to k$,

$$\frac{h \to k}{g \to k}\,k$$

induces the following expanded proof tree:

$$\cfrac{\cfrac{\text{some premise}}{\text{a first inference}}\,g \qquad \cfrac{h \to k}{g \to k}\,k}{\text{a second inference}}\,k$$

Thus $g \to k$ is only a valid composition provided that $k \to k$ also is. However, as we observed, $g \to k$ is the explicit premise in the sages' first attempted proof of $k \to k$. This proof would have been unacceptable, therefore, even if it had not exhibited the inherent circularity that motivates the sages' rejection of it, because of the circularity that it generates together with their proof of $g \to k$. Of course, the sages do reject the proof, for other closely allied reasons, so the present observation does not imply any oversight on their part.

In fact, no such circularity spans the eventually accepted proofs of $g \to k$ and $k \to k$, nor any combination of arguments that the sages finally agree to. This fact on its own, if any more evidence were needed, further supports the "hidden premise" reading of the *sugya*: because circularity is so easily encountered on that reading, the fact that the sages manage to avoid it certainly suggests that they were mindful of the threat.

A final type of apparent circularity should be addressed in order to be discounted. One outcome of the sugya is that no two rules are incomparable: *kal vachomer* is strictly stronger than *gezeirah shavah*, *gezeirah shavah* stronger than *hekeish*. But we observed in section four that this is also an assumption of the *kal vachomer* method. Thus, it seems, to begin using *kal vachomer* to determine the range of composition of these rules, one must know in advance a central fact about how the *sugya* will resolve. And indeed one can know this, the objection continues, because by deploying the *kal vachomer* method, one guarantees that all rules are in fact comparable. But this seems more like imposing an ordering on the rules than discovering one that is there.

The error in this charge of circularity is important to spot. The linearity of composition strength that the rules of sacred inference exhibit is not, after all, an assumption of the *kal vachomer* method. It is only an assumption of each application of *kal vachomer* to reason about the range of composition of the various inference rules. PRINCIPLES 1 and 2 merely indicate what standard of strength is being appealed to in designating one rule *kal*, another *chomer*. But any particular attempted *kal vachomer* inference could be refuted by pointing out that the rules being reasoned about in that inference are incomparable. Nothing about the use of *kal vachomer* to reason about itself and its fellow rules precludes that.

The value of modelling the principles underlying the sages' use of inference rules to reason about themselves is that doing so sheds light on what they take to be a relevant strength on which an inference can pivot. Even if what emerged from that analysis had been inconsistent, the sages' *method* would have been free of circularity and consistently applicable—appeals to multiple incompatible standards would merely have opened the way for *pirka* refutations of all attempts to reason at the meta-level so that this method would be worthless. The analysis uncovered, however, that the seemingly wide array of unrelated standards of inferential strength that the sages appeal to are in fact multiple faces of a single, coherent idea. In their able hands, this idea is put to strange and wonderful effect.

Acknowledgements

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APPENDIX: "*hekeish*, *gezeirah shavah*, *kal vachomer*" (*Zevachim* 49b–51a)

Remember this *sugya* through its key words: "*hekeish*, *gezeirah shavah*, *kal vachomer*."

A matter inferred through *hekeish* cannot return as a premise of a *hekeish*—[this principle has been established] either from Rava or from Ravina. What about returning as a premise of a *gezeirah shavah*? Can a matter inferred through *hekeish* do this?

Come listen. Rabbi Nasan ben Avtulmus says, "From where is it known that a garment on which a discoloration is found remains pure? The expressions '*korachat*' and '*gabachat*' are used concerning garments, and the expressions '*korachat*' and '*gabachat*' are used concerning man [regarding discolorations on his head]. Just as in the latter context a discoloration in one's entirety leaves one pure, so too in the present case a discoloration in the garment's entirety leaves it pure." [This inference is a *gezeirah shavah*.] And from where is the latter case inferred? From the verse *. . . from his head and to his feet . . .* which licenses a *hekeish* between "his head" and "his feet" as follows: Just as elsewhere [regarding one's body] becoming completely white and occurring over one's entirety leaves one pure, so too here [regarding one's head] becoming completely white and occurring over one's entirety leaves one pure.

Rabbi Yochanan responded: "Everywhere in the Torah we may draw an inference from something which itself has been inferred, outside the realm of the sacred, wherein we may not draw an inference from something which itself has been inferred. [I.e., this instance of a matter inferred through *hekeish* returning as a premise of a *gezeirah shavah* is not pertinent. Its examples are mundane, whereas our question must be understood as about composition of inferences in the realm of the sacred.] For if so, the Torah could just as well not mention that the *asham* sacrifice need be made in the north, and instead leave this to be inferred with a *gezeirah shavah* from the expression '*kadshei kadashim*' [here and also] in the context of the *chatat* sacrifice [where this law is established with the *hekeish* technique]. Does [the explicit injunction to make the sacrifice in the north] not tell us that a matter inferred through *hekeish* cannot return as a premise of a *gezeirah shavah*?"

[You might ask:] But perhaps [the text is as it is] because it is possible to invalidate [the alleged *gezeirah shavah* by posing this question]: In what way does the *asham* sacrifice compare to the *chatat* sacrifice, which unlike the *asham* atones for the most egregious guilt?

[But it happens that] two additional occurrences of "*kadshei kadashim*" are written, [strengthening the *gezeirah shavah* and overriding the dis-analogy.]

A matter inferred through *hekeish* can return as a premise of a *kal vachomer*—[this principle has been established] from what was taught in the house of Rabbi Yishmael. What about returning as a premise of a *binyan av*? Can a matter inferred through *hekeish* do this?

Rabbi Yirmyah said: "[If so] the Torah could just as well not mention that the *asham* sacrifice need be made in the north, and instead leave this to be inferred with a *binyan av* from the *chatat* sacrifice. What aspect of the Way is explicit mention of this requirement meant to reveal? Not to tell us that a matter inferred through *hekeish* cannot return as a premise of a *binyan av*?"

And by your lights [the Torah could just as well omit even the fact that the *chatat* sacrifice must be made in the north] and leave this too to be inferred with a *binyan av* from the *olah* sacrifice. What is the reason these inferences cannot be made? Because it is possible to invalidate [the alleged *binyan av* by posing this question]: In what way does the *chatat* sacrifice compare to the *olah* sacrifice, which unlike the *chatat* must be sacrificed in its entirety? Also possible is the following invalidation [of the alleged *binyan av* from] the *chatat* sacrifice: In what way does the *asham* sacrifice compare to the *chatat* sacrifice, which unlike the *asham* atones for the most egregious guilt?

Thus no one of these can be inferred from a single other. Can one be inferred from two others?

From which two would the inference be drawn? Have His Mercy not write as He has in the *olah* sacrifice verse, leaving its detail to be inferred from the *chatat* and *asham* sacrifices, and in what way does it compare to these which effect atonement? Have His Mercy not write as He has in the *chatat* sacrifice verse, leaving its detail to be inferred from the others, and in what way does it compare to these which must be male specimen? Have Him not write as He has in the *asham* sacrifice verse, leaving its detail to be inferred from the others, and in what way does it compare to these which pertain as much to the community as to the individual?

What about a matter inferred through *gezeirah shavah*? Can such return as a premise of a *hekeish*?

Rav Papa said: "From the verse *This is the law of the shelomim sacrifice . . . if for a todah . . .* we learn by *hekeish* that the *todah* sacrifice may come out of one's tithe from what we already know about the *shelamim* sacrifice, that it may come out of one's tithe. And about *shelamim* sacrifices themselves, from where do we know this? From the recurrence of the word '*sham*' [i.e., from a *gezeirah shavah*]."

Mar Zutra Brei D'rav Mari said to Ravina: "Tithed grain is entirely mundane [and thus

Rav Papa's example does not address our question about inference in the realm of the sacred.]"

Ravina replied to him: "Did he who spoke about the matter say that conclusions and premises alike must be sacred [for the restrictions on inference to apply]?"

What about a matter inferred through *gezeirah shavah*? Can such return as a premise of a *gezeirah shavah*?

Rami Bar Chama said, "It was taught: From the verse *... burnt flour ...* we see that the burnt loaf is made of flour. From where do we know the same of sacrificial loaves? The word '*chalot*' recurs, teaching so [through *gezeirah shavah*]. From where do we know the same of sacrificial wafers? The word '*matsot*' recurs, teaching so [through a second *gezeirah shavah* from the context of sacrificial loaves]."

Ravina replied to him: "What leads you to connect the word '*matsot*' with its recurrence in the context of the sacrificial loaves for this derivation? Perhaps it connects with another recurrence in the context of the sacrificial oven-loaf, allowing the same law to be derived [without thereby linking with a previous *gezeirah shavah*]."

However Rava said: "It was taught, from the verse *... he shall carry out both its entrails and its excrement ...* we see that the the high priest's sacrificial bull must be carried out intact. Lest one think that it must be burned intact as well, the words '*rosho*' and '*chera'av*' are stated here and again the words '*rosho*' and '*chera'av*' are stated elsewhere. [This engenders a *gezeirah shavah* teaching that] just as the sacrificial burning is preceded by dismemberment in that case, so too is it in the present case. Should one think in addition that just as the sacrificial burning is preceded by skinning in that case, so too must it be in the present case, *... its entrails and its excrement ...* is taught."

What is being taught exactly?

Rav Papa said: "As the excrement is in its entrails, so the meat should be in the skin."

And it was taught: Rebbi says, "the words '*or*' and '*basar*' and '*feresh*' are stated here [in a verse about the Day of Atonement sacrifices] and the words '*or*' and '*basar*' and '*feresh*' are stated there. Just as burning is preceded by dismemberment without skinning in that case, similarly must it be in the present case."

What about a matter inferred through *gezeirah shavah*? Can such return as a premise of a *kal vachomer*?

Let us address this question with a *kal vachomer*. Seeing as that *hekeish*, which cannot lead into a *hekeish*—[this principle having been established] either from Rava or from

Ravina—can lead into a *kal vachomer*—[this having been established] from what was taught in the house of Rabbi Yishmael—can it not be inferred that *gezeirah shavah*, which can lead into a *hekeish*—as Rav Papa established—can lead into a *kal vachomer*?

This makes sense to anyone who follows Rav Papa. But what can be said to someone who does not follow Rav Papa?

Then let us consider a different application of *kal vachomer*. Seeing as that *hekeish*, which cannot lead into a *hekeish*—[this principle having been established] either from Rava or from Ravina—can lead into *kal vachomer*—[this having been established] from what was taught in the house of Rabbi Yishmael—can it not be inferred that *gezeirah shavah*, which can lead into a fellow *gezeirah shavah*—as Rami Bar Chama established—can lead into a *kal vachomer*?

What about a matter inferred through *gezeirah shavah*? Can such return as a premise of a *binyan av*?

Let this remain a question.

What about a matter inferred through *kal vachomer*? Can such return as a premise of a *hekeish*?

Let us address this question with a *kal vachomer*. Seeing as that *gezeirah shavah*, which cannot pick up from a *hekeish*—as Rabbi Yochannon established—can lead into a *hekeish*—as Rav Papa established—can it not be inferred that *kal vachomer*, which can pick up from a *hekeish*—[this having been established] from what was taught in the house of Rabbi Yishmael—can lead into a *hekeish*?

This makes sense to anyone who follows Rav Papa. But what can be said to someone who does not follow Rav Papa?

Let this remain a question.

What about a matter inferred through *kal vachomer*? Can such return as a premise of a *gezeirah shavah*?

Let us address this question with a *kal vachomer*. Seeing as that *gezeirah shavah*, which cannot pick up from a *hekeish*—as Rabbi Yochannon established—can lead into a *gezeirah shavah*—as Rami Bar Chama established—can it not be inferred that *kal vachomer*, which can pick up from a *hekeish*—[this having been established] from what was taught in the house of Rabbi Yishmael—can lead into a *gezeirah shavah*?

What about a matter inferred through *kal vachomer*? Can such return as a premise of a *kal vachomer*?

Let us address this question with a *kal vachomer*. Seeing as that *gezeirah shavah*, which cannot pick up from a *hekeish*—as Rabbi Yochannon established—can lead into a *kal vachomer*—as we just said—can it not be inferred that *kal vachomer*, which can pick up from a *hekeish*—[this having been established] from what was taught in the house of Rabbi Yishmael—can lead into a *kal vachomer*?

And this very *kal vachomer* is a son of a *kal vachomer*.

A son of a son of a *kal vachomer*!

Then let us consider a different application of *kal vachomer*. Seeing as that *hekeish*, which cannot pick up from a *hekeish*—[this principle having been established] either from Rava or from Ravina—can lead into *kal vachomer*—[this having been established] from what was taught in the house of Rabbi Yishmael—can it not be inferred that *kal vachomer*, which can pick up from a *hekeish*—[again] from what was taught in the house of Rabbi Yishmael—can lead into a *kal vachomer*?

And this very *kal vachomer* is a son of a *kal vachomer*.

What about a matter inferred through *kal vachomer*? Can such return as a premise of a *binyan av*?

Rabbi Yirmyah said: "Come listen. Should one perform the sacred rite on a sacrificial bird and find it to be unfit for sacrifice: Rabbi Meir says that it does not transmit impurity if eaten; Rabbi Yehudah says it does transmit impurity if eaten. Rabbi Meir said, 'let us address this matter with a *kal vachomer*. With livestock, though their offal transmit impurity if touched or if carried, their mundane slaughter rids them, if they are found to be unfit, of impurity. Can it not be inferred that the mundane slaughter of fowl, since their offal do not transmit impurity if touched or if carried, rids them, if they are found to be unfit, of impurity. [Now a *binyan av* is possible:] Just as we find with mundane slaughter that it effects fitness for consumption [if it is found to be so fit] and rids one found not to be fit or impurity, also the sacred rite effects fitness for consumption and rids an unfit specimen of impurity.' Rabbi Yosei says, 'The analogy with the offal of livestock is enough, in so far as it establishes that a bird's mundane slaughter rids it of impurity. It cannot establish that its sacred rite does.' "

But this isn't [a legitimate attempt at a *kal vachomer* leading into a *binyan av*]. See over there. [The classical commentaries understand this to be a reference to folio 69, where the second step of Rabbi Meir's inference is shown to be a *hekeish*] And were it so, it would evidently proceed from a case of slaughter that is mundane [thereby failing to be an undisputed instance of composition of inferences in the realm of the sacred.]

What about a matter inferred through *binyan av*? Can such return as a premise of a *hekeish* or *gezeirah shavah* or *kal vachomer* or *binyan av*?

Answer from here one of these questions: "Why did they say that sacrificial blood left until after the proper time for its ritual application is allowed [to be left on the altar if erroneously put there]?—Because sacrificial parts left until after the proper time for their ritual application are so allowed. That sacrificial parts left until after the proper time for their ritual application are fit, we know, because the meat of the sacrifice, if left until after the proper time of its ritual, is fit. And something taken out of the appropriate premises?—Because such is fit as a sacrifice on a transient altar. Something that has become impure?—Because such are fit in the case of communal sacrifices. Something that had its initial rites performed for the purpose of having its sacrifice finalized after its proper time?—Because its initial rites brought it partially into the realm of the sacred under the classification of '*pigul*.' If its initial rites were performed for the purpose of having its sacrifice finalized outside of the appropriate premises?—From a *hekeish* with '*pigul*.' If someone disqualified to do so gathered its blood after it was slaughtered or applied its blood to the altar?—[Such acts are fit in retrospect of their occurrence even in the case of an individual's sacrifice, despite being initially,] in the case of those who are nevertheless qualified to perform these rites in the case of communal sacrifices."

But may we draw an inference regarding something done improperly from something done properly? The Sage who taught this supported his teaching principally from the verse *This is the law of the olah sacrifice . . .* [and issued the point by point explanations enumerated above only for the sake of clarification.]