Philosophy of Logic
The Concept of Completeness

CONTACT INFORMATION

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THIS COURSE

The desire to fully individuate and comprehensibly codify the modes of reason or categories of thought has played central but shifting roles in the history of philosophy and science. This completeness concept attained its mature, scientific form in Kurt Gödel's *1929* thesis, where the idea of a purely formal logical system being fully adequate to the subject matter it was designed to encode was ratified with a rigorous mathematical proof. This result ushered in a paradigm of scientific research: with the fields of proof-theory and model-theory being clearly delineated and the relationship between them well understood, logic became a branch of modern mathematics.

The completeness theorem of *Gödel 1929* is in a sense elementary, its proof easily conveyed to non-experts. The question that the theorem settles is, moreover, natural and at the heart of logical thinking. For these reasons historians have expressed wonder as to how it took so long for experts to discover its proof. The answer to this question is that prior to the work of Gödel, and even for many years afterwards, almost no one thought of logical completeness as we do today. Yet his concept has such a grip on us today that we struggle to understand the rival, or complimentary, notions that logicians have had. What were these ways of thinking about completeness, what were their merits, and how did the prevailing concept of logical completeness emerge from them? Our interest is to understand the network of ideas that preceded the prevailing concept, in order to appreciate that even asking the question that Gödel answered was a major intellectual achievement.

TEXTS

Anticipated reading will be from the following sources. Some of these texts will be studied in full detail; others will only be read in excerpt. Many of these books would be quite expensive to acquire, so my plan is just to distribute them prior to each class. Our main focus will be on texts marked

†. Other texts will serve either as commentaries on these or in filling in the historical context.

Aho, T. and M. Yrjönsuuri. 2009. "Late medieval logic." L. Haaparanta (ed.) *The Development of Modern Logic*.

†Aristotle, c. 350BCE. *Prior Analytics* Book I.

Awodey, S. and E. H. Reck. 2002. "Completeness and categoricity, part I: 19th Century axiomatics to 20th Century metalogic." *History and Philosophy of Logic* **23**(1): 1–30.

Bolzano, B. 1804. *Betrachtungen über einige Gegenstände der Elementargeometrie*. Prague: Karl Barth. Translated in part by S. Russ in *Ewald 1996*, 172–4.

†Bolzano, B. 1810. *Beyträge zu einer begründeteren Dartellung der Mathematik*. Prague: Caspar Widtmann. Trans. by S. Russ in *Ewald 1996*, 174–224.

Bolzano, B. 1817. *Rein analytischer Beweis des Lehrsatzes, dass zwischen je zwei Werten, die ein entgegengesetztes Resultat gewähren, wenigstens eine reelle Wurzel der Gleichung liege*. Prague: Gottlieb Haase. Trans. by S. Russ in *Ewald 1996*, 225–48.

†Bolzano, B. 1837. *Wissenschaftslehre. Versuch einer ausführlichen und grössenteils neuen Darstellung der Logik mit steter Rücksicht auf deren bisherigen Bearbeiter.* Sulzbach: Seidel. Trans. by B. Terrel in B. Bolzano *Theory of Science*, J. Berg (ed.) 1973. Boston: D. Reidel Publishing Company.

Brouwer, L.E.J. 1928. "Mathematics, science, and language," in *Ewald 1996* vol. II, 1170–1185.

Corcoran, J. 1972. "Completeness of an Ancient Logic," *Journal of Symbolic Logic* **37** (4), 696-702.

De Morgan, A. 1842. "On the foundation of algebra," reprinted in *Ewald 1996*.

Dreben, B. and J. van Heijenoort. 1986. "Introductory note to *Gödel 1929*, *1930* and *1930b* in CW vol I.

Franks, C. 2010. "Cut as consequence," *History and Philosophy of Logic* **31**(4), 349–79.

Franks, C. 201(?). "Logical Completeness, form, and content: an archaeology," in J. Kennedy (ed.) *Interpreting Gödel: Critical Essays.*

†Gentzen, G. 1932. "Über die Existenz unabhängiger Axiomensysteme zu unendlichen Satzsystemen," *Mathematische Annalen* **107**, 329-50. Translated as "On the existence of independent axiomsystems for infinite sentence systems" in *Szabo 1969*, 29–52.

†Gentzen, G. 1934–35. "Untersuchungen über das logische Schliessen." Gentzen's doctoral thesis at the University of Göttingen, translated as "Investigations into logical deduction" in *Szabo 1969*, 68–131.

George, R. 1986. "Bolzano's concept of consequence," *the Journal of Philosophy.*

†Gödel, K. 1929. "Über die Vollständigkeit des Logikkalküls," Gödel's doctoral thesis at the University of Vienna, translation by S. Bauer-Mengelberg and Jean van Heijenoort as "On the completeness of the calculus of logic" reprinted in CW vol I. 60–101.

Gödel, K. 1930. "Die Vollständigkeit der Axiome des logischen Functionenkalküls," *Monatshefte für Mathematik und Physik* **37**, 349-60, translation by S. Bauer-Mengelberg as "The completeness of the axioms of the functional calculus of logic" reprinted in CW vol I. 102–23.

†Gödel, K. 1930a: "Lecture on completeness of the functional calculus," in CW vol III.

Goldfarb, W. "Introductory note to *Gödel 1930a.*"

Henkin, L. 1949. "The completeness of the first-order functional calculus." *Journal of Symbolic Logic* **14**, 159–66.

Hilbert, D. 1929. "Probleme der Grundlegung der Mathematik," *Mathematische Annalen* **102**, 1–9.

Kennedy, J. 2011. "Gödel's thesis: An appreciation." M. Baaz (ed.) *Horizons of Truth: Kurt Gödel and the Foundations of Mathematics*, 95–110.

Locke, J. 1690. *An Essay Concerning Human Understanding.*

Moore 1988. "The emergence of first-order logic," in *Aspray and Kitcher 1988*, 95–135.

Peirce, C. S. 1885. "On the algebra of logic: a contribution to the philsophy of notation," in *Ewald 1996*, 608–31.

Post, E. 1921. "Introduction to a general theory of propositions," reprinted in *van Heijenoort 1967a*, 265–82.

Smiley, T. 1994. "Aristotle's Completeness Proof," *Ancient Philosophy* **14**, 25-38.

Sundholm, G. 2009. "A century of judgement and inference, 1837–1936," L. Haaparanta (ed.) *The Development of Modern Logic*, 263–316.

Van Heijenoort, J. 1967. "Logic as language and logic as calculus," *Synthese* **17**, 324–30.

Vilkko, R. "The logic question during the first half of the 19th Century," L. Haaparanta (ed.) *The Development of Modern Logic*. 203–221.

Zach, R. 1999. "Completeness before Post: Bernays, Hilbert, and the development of propositional logic," *Bulletin of Symbolic Logic* **5**, 331–66.

REQUIREMENTS

Students seeking a grade are asked to write an essay of historical, mathematical, or philosophical significance. My joints are pretty loose: you may be creative with the sort of project you pursue, and I will be happy to appreciate it according to its merits without any preconceptions of what a good term paper should be like. A good rule of thumb is to try to write an essay that a reasonable person could be expected to want to read a second time.

NOTE

Please be aware of the University's policies regarding academic honesty, anti-discrimination, and access to education for students with disabilities.

Here is the web-page of the office for students with disabilities:

`http://www.nd.edu/~osd/NEWHOMEPAGE.htm`

Here is the Philosophy Department's web-page devoted to academic honesty, with links to information about plagiarism and the University's honor code:

`http://philosophy.nd.edu/undergraduate-program/honesty/`

In addition I am someone you can approach if you have concerns about discrimination or proper scholarly behavior, whether or not the concern is related to this course.