Center for Mathematics at Notre Dame

INTERNATIONAL CONFERENCE ON BOUNDARIES AND ERGODIC GEOMETRY

Monday June 1 – Friday June 5, 2015

University of Notre Dame

ABSTRACTS

All talks will be held in 127Hayes-Healy and Coffee Breaks/Registration in 257 Hurley

---

Yves Benoist, Université Paris-Sud

On the regularity of stationary measures

We will construct on any semisimple Lie group a finitely supported and Zariski dense probability measure whose stationary measure on the flag variety has a differentiable density. This joint work with JF. Quint extends a recent result of J. Bourgain.

Francis Bonahon, University of Southern California

Hitchin representations, surface foliations, and pseudo-Anosov surface homeomorphisms

Hitchin representations of a surface group are higher rank analogues of hyperbolic metrics on the corresponding surface. They form a whole component of the character variety of homomorphisms from the surface group to PSL_n(R). I will describe a parametrization of this Hitchin component that is well adapted to a foliation of the surface with saddle-type singularities. I will then give a brief application to the dynamics of the action of a pseudo-Anosov homeomorphism of the surface on the Hitchin component.

Marc Bourdon, Université Lille 1

Hyperbolic spaces and conformal dimension

The conformal dimension of the boundary of a (Gromov) hyperbolic space is a quasi-isometry invariant of the space. It is bounded by below by the topological dimension of the boundary, and by above by its Hausdorff dimension. In this talk, we will present the conformal dimension, and we will be interested in determining the hyperbolic groups such that the conformal dimension of their boundary is equal to the topological dimension.

Aaron Brown, University of Chicago

Stiffness of stationary measures for surface diffeomorphisms and geometric criterion for the vanishing of the Furstenberg-Kullback-Ledrappier entropy.

(Joint with Federico Rodriguez-Hertz.) Consider a fixed surface M, a set of volume preserving diffeomorphisms of M, a probability measure on the set of diffeomorphisms and a stationary measure for the random walk. Our main technical theorem is that for hyperbolic stationary measures such that no measurable line field is preserved a.s. we show that the measure is either finitely supported or a component of volume. The hypotheses hold for perturbations of algebraic systems and allow us to classify all stationary measures for such systems.

In one case in the proof, we derive stiffness of a stationary measure by showing the vanishing of the Furstenberg—Kullback—Ledrappier entropy under certain geometric conditions. We conjecture a more general result is true: that the Furstenberg—Kullback—Ledrappier entropy coincides with the difference between entropy contribution and conditional entropy coming from negative Lyapunov exponents.

Clark Butler, University of Chicago

Rigidity of the Lyapunov spectrum for geodesic flows on negatively curved symmetric spaces

We study the relationship between the Lyapunov spectrum of the geodesic flow of a closed negatively curved manifold and the geometry of the manifold. We show that if the Lyapunov exponents with respect to a Gibbs measure of the geodesic flow acting on expanding horospheres are all equal then the manifold has constant negative curvature. We then formulate an analogous condition (inspired by the conformal geometry of the Heisenberg group) on the Lyapunov spectrum of a perturbation of the symmetric metric on a complex hyperbolic manifold and prove that the metric is locally symmetric as well.

Pierre-Emmanuel Caprace, Université Catholique de Louvain

Boundaries of simple locally compact groups

This talk, based on joint work with Colin Reid and George Willis, is devoted to the class of compactly generated simple locally compact groups as a whole. I will explain how the investigation of the local structure of a group G in that class gives rise to the construction of a compact G-space which happens to be a G-boundary. This provides a dynamical tool that helps in elucidating the algebraic properties of G.

Yves Guivarc'h, Université de Rennes 1

Boundary theory and spectral gap properties on spaces of Holder functions. Some consequences in extreme value theory for affine random walks.

We describe a spectral gap property for twisted random walks on projective spaces, and we give some consequences for affine random walks: analogues of Fréchet's law and Sullivan's logarithm law. We sketch a proof based on Kac's return theorem and on Wiener-Ikehara's tauberian theorem.

Ursula Hamenstädt, Rheinische Friedrich-Wilhelms - Universität Bonn

Random walks and hyperbolic groups with sphere boundary

The Gromov Thurston Mostow states the following. Let M,N be closed hyperbolic manifolds of the same dimension n. If there is a degree one map from M to N and if the volumes of M,N coincide then the fundamental groups of M,N are isomorphic (and M,N are isometric if n is at least three).

We extend this result to the case when N is allowed to be a $K(G,1)$ of a torsion free hyperbolic group whose boundary is a sphere of dimension n-1 and where volume means simplicial volume.

Camille Horbez, Université de Rennes 1

Horoboundary of outer space and growth under random automorphisms

I will present results describing the growth of a nontrivial conjugacy class in the free group F_N under random products of outer automorphisms of F_N. These rely on a description of the horoboundary of Culler-Vogtmann's outer space.

Vadim Kaimanovich, University of Ottowa

Stopping times and Poisson boundaries

The Poisson boundary of a random walk on a group is defined as the space of ergodic components of the time shift in its path space. In this talk I will explain why Markov stopping times applied to the path space (in a way similar to time changes in the classical dynamical setup) give rise to new random walks with the same Poisson boundary. Other related questions will also be discussed.

Ilya Kapovich, University of Illinois – Urbana/Champaign

Dynamics and polynomial invariants of free-by-cyclic groups

The theory developed by Thurston, Fried and McMullen provides a near complete picture of the various ways a hyperbolic 3-manifold M can fiber over the circle. Namely, there are distinguished convex cones in the first cohomology $H^1(M;R)$ whose integral points all correspond to fibrations of M, and the dynamical features of these fibrations are all encoded by McMullen's "Teichmuller polynomial." This talk will describe recent work developing aspects of this picture in the setting of a free-by-cyclic group G. Specifically, we will describe a polynomial invariant that determines a convex polygonal cone C in the first cohomology of G whose integral points all correspond to algebraically and dynamically interesting splittings of G. The polynomial invariant additionally provides a wealth of dynamical information about these splittings. This is a joint work with Spencer Dowdall and Christopher Leininger.

Anders Karlsson, Université de Genève

Noncommuting Random Products

The title is borrowed from a seminal paper by Furstenberg from 1963. In the introduction of that paper he raises the question whether there is a limit law, similar to the law of large numbers, for products\[X_{1}X_{2}X_{3}...X_{n}\] as $n\rightarrow\infty$, where X_{i} are transformations chosen at random. Such products appear in several contexts within mathematics as well as in other sciences. The transformations could be bounded linear operators, holomorphic maps or just elements in an abstract group. I will discuss one answer to this general question using metric spaces, their functionals, and subadditive ergodic theory. Based on joint work with S. Gouëzel extending an earlier joint work with F. Ledrappier.

Steve Lalley, University of Chicago

Local Limit Theorems and Boundary Theory for Random Walks on Discrete Groups

By a "local limit theorem" we mean a sharp asymptotic formula for the probability of return to the starting point after a large number of steps. We will discuss recent progress in obtaining such theorems for random walks on several classes of discrete groups, notably hyperbolic groups and Cartesian products of hyperbolic groups with other groups. We will also exhibit connections with the structure of the Martin boundaries of such walks.

Seonhee Lim, Seoul National University

Entropy, energy and measures on the boundary

The talk will be about two joint works with François Ledrappier: one on the entropy of hyperbolic buildings and the other on Brownian motion in negatively curved manifolds. We will explain how both problems are related to families of measures on the boundary.

Mark Pollicott, University of Warwick

The Kusuoka Measure for the Sierpinski Triangle

The Kusuoka measure plays an important role in defining the Laplacian on the Sierpinski Triangle. However, remarkably little is known about its ergodic properties. We show that Kusuoka measure is strong mixing with an exponential rate for the natural shift. This is joint work with A.Johansson and A. Oberg.

Giulio Tiozzo, Yale University

Random walks on weakly hyperbolic groups and applications

Let us consider a group G of isometries of a delta-hyperbolic metric space X, which is not necessarily proper (e.g. it could be a locally infinite graph). We can define a random walk by picking products of random elements of G, and projecting this sample path to X.

We show that such a random walk converges almost surely to the Gromov boundary of X, and with positive speed.

As an application, one can identify the Poisson boundary of acylindrically hyperbolic groups. Moreover, one proves that a random k-generated subgroup of the mapping class group is convex cocompact, and a similar statement holds for Out(F_n).

This is joint work, partly with J. Maher and partly with S. Taylor.

Zhiren Wang, Pennsylvania State University

Global Rigidity of Anosov Actions by Higher Rank Lattices

We will discuss a recent work with Aaron Brown and Federico Rodriguez Hertz on smooth classification of Anosov actions by higher rank lattices on nilmanifolds. In particular, we will explain how the existence of an Anosov diffeomorphism from the group action leads to Anosov property of generic elements in the acting group, allowing to make use of large abelian subgroups.

Wolfgang Woess, Technische Universität Graz

Quasi-isometries, harmonic functions and boundaries of DL-graphs, Sol, and treebolic spaces

In this talk, I will review the story of the construction of transitive graphs that are not quasi-isometric with Cayley graphs. They are related with lamplighter groups, and analogous non-discrete structures are the Sol-manifolds (resp. -groups) and treebolic spaces. The main focus will be on random walks, resp. Brownian motions on those structures and the associated harmonic functions.

Alex Wright, Stanford University

The boundary of a GL(2,R) orbit closure.

We consider GL(2,R) orbit closures of translation surfaces. These orbit closures are submanifolds of moduli spaces described by linear equations in local coordinates. We describe the boundary such submanifolds. The proofs involve the dynamics of the horocycle flow and flat geometry. One concrete application will be given, and we will also hint at the promise for many other applications. Joint work in progress with Maryam Mirzakhani.