Center for Mathematics at Notre Dame

THEMATIC PROGRAM ON BOUNDARIES AND DYNAMICS

Undergraduate Mini-Course Week

Tuesday May 19 – Friday May 22, 2015

University of Notre Dame

LECTURE ABSTRACTS

All talks will be held in 229 Hayes-Healy and Coffee Breaks/Registration in 257 Hurley

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Diana Davis (Northwestern University)

Lines in Positive Genus: An Introduction to Flat Surfaces

If we identify oppositely-oriented parallel edges of a polygon or collection of polygons, we obtain a flat surface. In this course, we discuss geodesics on such surfaces, their associated cutting sequences, the connection to continued fractions, the automorphisms of the surfaces, and some families of so-called Veech surfaces that have many automorphisms. This course will be geometric and example-driven, with lots of pictures.

Pablo Lessa (Montevideo)

Recurrence vs Transience: An Introduction to Simple Random Walks on Discrete Structures

Pólya's theorem states that a simple random walk on an infinite 2d grid will visit every vertex infinitely many times, but on a 3d grid it will eventually leave any finite set of vertices never to return. In short the random walk on the 2d grid is recurrent, but on the 3d grid it is transient.

In these lectures we investigate the question of understanding which infinite graphs admit recurrent random walks. After a series of examples and brief introduction to probability theory we concentrate on two families of graphs: Trees and Cayley graphs.

A beautiful result due to Terry Lyons characterizes the recurrence or transience of an infinite graph in terms of the capacity of the graph to transport water when one imagines that all edges are pipes of the same size. Using this result we obtain some characterizations of recurrent trees which are due to Benjamini, Peres, and R. Lyons.

In order to characterize recurrent Cayley graphs we are lead to discuss the growth properties of groups, isoperimetric inequalities, and Gromov's theorem on groups of polynomial growth.

We hope the lectures will serve as a motivation for the study of random walks in a wide variety of contexts. We strive to provide a starting point for the interested student to begin his journey into the research literature.

Roland Roeder (Indiana University/Purdue University Indianapolis)

Around the Boundary of Complex Dynamics

We introduce the exciting field of complex dynamics at an undergraduate level while reviewing, reinforcing, and extending the ideas learned in a typical first course on complex analysis. Julia sets and the famous Mandelbrot set will be introduced and interesting properties of their boundaries will be described. We will conclude with a discussion of problems at the boundary between complex dynamics and other areas, including a nice application of the material we have learned to a problem in astrophysics.

Bryce Weaver (Indiana University - Bloomington)

Margulis Counting Formula for Geodesic Flows over Hyperbolic Surfaces

We will define basic notions of a dynamical system, in particular a continuous flow. We will discuss some significant structure that can arise, namely the stable and unstable sets (manifolds). We will then consider the case of SL(2,R) with the flow coming from the diagonal group. The notions stable and unstable manifolds will be made concrete in this setting. We will then discuss the idea of periodic orbits and its relationship to complexity of a system (which will be defined via entropy, but mastery of this topic is not expected). We will close with an approximate proof of the Margulis counting formula in a particular case. We will assume that the students are comfortable with matrices and multi-variable calculus.