Center for Mathematics at Notre Dame

INTERNATIONAL CONFERENCE IN COMMUTATIVE ALGEBRA AND
ITS INTERACTION WITH ALGEBRAIC GEOMETRY

Sunday June 16 – Friday June 21, 2019
University of Notre Dame

TALK ABSTRACTS

Talks listed in alphabetical order by speaker name.

Lucho Avramov (University of Nebraska)
Monoids of Betti tables over short Gorenstein algebras

A standard graded algebra R is said to be short if $R_i = 0$ for $i \neq 0,1,2$. In case R is Gorenstein, an explicit computation of the monoid of Betti tables of R-modules will be presented. The result provides new information on the representation theory of R. Some of the techniques developed for its proof apply in more general contexts.

The talk is based on recent joint work with Courtney Gibbons and Roger Wiegand.

Giulio Caviglia (Purdue University)
Upper bounds for Betti numbers in Hilbert schemes of complete intersections.

Let S be a polynomial ring over a field and I a homogeneous ideal containing a regular sequence of known degrees. I will discuss the existence of sharp upper bounds for the Betti numbers of I in terms of the degrees of the regular sequence and the Hilbert function, or the Hilbert polynomial, of I. This is a joint work with Alessio Sammartano.

Marc Chardin (Sorbonne Université, France)
Complete intersections in a product of projective spaces

One could define a complete intersection in some irreducible scheme X as a subscheme defined by as many equations as its codimension. Whenever X is a projective space, this corresponds to subschemes defined by homogeneous complete intersection ideals; as a consequence many homological invariants are entirely determined by the degrees of these equations. In a product of projective spaces, the situation is quite different. We will illustrate this on two examples : a hypersurface and the study of complete intersection sets of points in a product of two projective spaces. This is based on ongoing joint work with Navid Nemati.

Aldo Conca (University of Genoa, Italy)
Ideals associated to graphs and special sections of determinantal varieties

According to Lovasz, Saks and Schrijver an orthogonal d-dimensional representation of a graph G is a map from the vertices of G to the real d-dimensional space sending non-edges to orthogonal vectors. The set of all the orthogonal d-dimensional representations of G is a variety whose defining ideal, the Lovasz-Saks-Schrijver ideal of G, is generated by the quadrics expressing the orthogonality conditions among non-edges. For which values of d is the Lovasz-Saks-Schrijver ideal of G radical, prime or complete intersection? These are the questions we will discuss in the talk. Furthermore we will show how this study is related to the investigation of special sections of determinantal varieties. For example we show that in characteristic 0 the ideal of 3-minors of a generic matrix with some entries replaced by 0 is always reduced and that the analogous statement for 4-minors does not hold in general.

The talk is based on a joint paper with Volkmar Welker (Marburg).

Dale Cutkosky (University of Missouri)
Mixed Multiplicities of Filtrations

We develop a theory of mixed multiplicities for possibly non Noetherian filtrations of m-primary ideals.

We show that many of the classical theorems for mixed multiplicities of m-primary ideals are true in this more general situation. An example which we will discuss is of filtrations associated to m-valuations, which are particularly nice in this theory, even though they are generally not Noetherian. This talk will include joint work with Parangama Sarkar, Hema Srinivasan and Jugal Verma.

Hai Long Dao (University of Kansas)
Loaded Modules

Over a commutative Noetherian local ring R, we will study the notion of loaded modules. These objects can be shown to include Ulrich modules and integrally closed ideals, and still hold many well-known desirable properties.

David Eisenbud (MSRI & University of California Berkeley)
Residual Intersections for Chasles to Ulrich

I'll briefly sketch the history of residual intersection theory from the 3264 conics of Chasles through the work of Fulton, Artin-Nagata and Huneke; and then concentrate on the work of Bernd Ulrich and his co-authors on the question of when a residual intersection is Cohen-Macaulay (and refinements).

Louiza Fouli (New Mexico State University)
The Core of Ideals

The core of an ideal in a Noetherian local ring with infinite residue field is defined as the intersection of all its reductions and encodes information about all possible reductions. In the first part of the talk we will give a brief survey of this topic highlighting the contributions of Professor Bernd Ulrich. In the second part of the talk we will discuss some new results concerning the core of monomial ideals. This is joint work with Jonathan Monta\~{n}o, Claudia Polini, and Bernd Ulrich.

Mel Hochster (University of Michigan)
Faithfulness of Top Local Cohomology

Let R be a local domain and I a proper ideal of R. Let d be the cohomological dimension of I, so that $H^d\_I(R)$ is not 0 but all higher local cohomology modules vanish. It is an open question, raised in the thesis of L. Lynch, whether $H^d\_I(R)$ must be a faithful R-module. The talk will discuss the status of this question and report on joint work Jack Jeffries which proves faithfulness when the arithmetic rank of I is d *provided that* R has characteristic $p > 0$. So far as we know, the question remains open when I has arithmetic rank d and R contains the rational numbers.

Craig Huneke (University of Virginia)
The Mathematical Work of Bernd Ulrich

This talk will survey some of the many highlights of Bernd's mathematics, ranging through Berger's conjecture, linkage, residual intersections, Gorenstein rings, Rees algebras, integral closures, the core of ideals, and equisingularity.

Srikanth Iyengar (University of Utah)
Modules with no higher self-Tors

The following question was raised, implicitly, by Liana Şega:
If a finitely generated module M over a local ring R satisfies Tor_i(M,M)=0 for i>>0, is then M of finite projective dimension? I will report on progress on this question, made in collaboration with Avramov, Nasseh, and Sather-Wagstaff.

Steve Kleiman (MIT)
Macaulay Duality over Any Base

Traditionally, Macaulay Duality furnishes a useful canonical bijective correspondence between Artinian quotients of a polynomial ring over a base field and modules of linear functionals on forms. In joint work in progress with Jan Kleppe of Oslo, this duality is generalized over an arbitrary Noetherian base ring, thus providing a suitable framework for studying a family of Artinian quotients by investigating its dual family.

Andrew Kustin (University of South Carolina)
Resolutions which are Differential Graded Algebras

Let P be a commutative Noetherian ring and F be a self-dual acyclic complex of finitely generated free P-modules. Assume that F has length four and F_0 has rank one. We prove that F can be given the structure of a Differential Graded Algebra with Divided Powers; furthermore, the multiplication on F exhibits Poincare Duality. This result is already known if P is a local Gorenstein ring and F is a minimal resolution. The purpose of the present project is to remove the unnecessary hypotheses that P is local, P is Gorenstein, and F is minimal.

Linquan Ma (Purdue University)
Asymptotic Lech’s inequality

In 1960 Lech proved a simple inequality that relates the Hilbert-Samuel multiplicity and the colength of an m-primary ideal in a Noetherian local ring. However, this inequality is almost never sharp. In this talk, we discuss optimal versions of Lech's inequality for sufficiently deep ideals. We prove them using Hilbert-Kunz theory in characteristic $p>0$, and we conjecture that they hold in arbitrary characteristic. This is joint work with Craig Huneke, Pham Hung Quy, and Ilya Smirnov.

Claudia Miller (Syracuse University)
Betti numbers of the Frobenius powers of the maximal ideal over generic hypersurfaces in 3 variables.

We discuss the conjectured curious behavior of the Betti numbers of the Frobenius powers of the maximal ideal in hypersurfaces $R=k[x,y,z]/(f)$, where k is a field of positive characteristic p, as described by Kustin and Ulrich. We prove that if f is chosen suitably generically (with k uncountable) and has degree of opposite parity to p then high enough Frobenius powers of the maximal ideal have identical graded Betti numbers up to explicit shifts. We also find the Hilbert-Kunz function and some structural features of the resolution and its associated matrix factorization. The proof is based on a technical result on the growth of the socles in the quotients, which in turn is based on our constructions of free resolutions of relatively compressed Artinian graded algebras. This is joint work with Hamid Rahmati and Rebecca R.G.

Viet Trung Ngo (Vietnam Academy of Science and Technology)
Depth functions of powers of homogeneous ideals

A classical result of Brodmann says that the numerical non-negative function depth R/I^t, where I is a homogeneous ideal in a polynomial ring R, is always convergent, i.e. it is constant for t large enough. In 2005, Herzog and Hibi posed the conjecture that the function depth R/I^t can be any numerical non-negative convergent function. This talk will will give an affirmative answer to this conjecture and shows a similar result for the symbolic powers $I^{(t)}$ of I, namely that for any numerical positive function f(t) which is periodic for t large enough, there is a homogeneous ideal I in a polynomial ring R such that depth $R/I^{(t)} = f(t)$ for $t > 0$. It is still an open question whether the function depth $R/I^{(t)}$ is always periodic for t large enough.

Irena Peeva (Cornell University)
Regularity of prime ideals

This talk will discuss some natural questions about the regularity of prime ideals. We give counterexamples to the Eisenbud-Goto Regularity Conjecture using Rees algebras, and also counterexamples that do not rely on the Mayr-Meyer construction. Furthermore, examples of prime ideals for which the difference between the maximal degree of a minimal generator and the maximal degree of a minimal first syzygy can be made arbitrarily large are provided. Using a recent result of Ananyan-Hochster we show that there exists an upper bound on regularity of prime ideals in terms of the multiplicity alone. This is joint work with Giulio Caviglia, Marc Chardin, Jason McCullough, and Matteo Varbaro.

Marilina Rossi (University of Genoa, Italy)
The Castelnuovo-Mumford regularity of the “blow-up algebras”

The Castelnuovo-Mumford regularities of the Rees algebra, of the associated graded ring and of the fiber ring of an ideal are strongly related. We present a survey on this topic and we show how the stability of the Ratliff-Rush filtration and the invariance of reduction number of the ideal give interesting consequences on the regularity of these graded algebras. In the case of homogeneous ideals generated in the same degree these methods recently led to a counterexample to a conjecture stated by D. Eisenbud and B. Ulrich.

Anurag Singh (University of Utah)
Differential operators on some invariant rings

Work of Levasseur and Stafford describes the rings of differential operators on various classical invariant rings of characteristic zero; in each of these cases, the differential operators form a simple ring. Towards an attack on the simplicity of rings of differential operators on invariant rings of linearly reductive groups over the complex numbers, Smith and Van den Bergh asked if reduction modulo p works for differential operators in this context. In joint work with Jack Jeffries, we establish that this is not the case for certain SL-invariants.

Karen Smith (University of Michigan)
Cluster Algebras in Prime Characteristic

Cluster Algebras are commutative algebras introduced by Sergey Fomin and Andrei Zelevinsky in order to capture some of the unifying ideas occurring throughout combinatorics, representation theory and total positivity. Defined by a simple combinatorial process, the commutative algebraic properties of cluster algebras have not yet been deeply studied. In this talk, I will introduce cluster algebras, show that upper cluster algebras are always Frobenius split, and that locally acyclic cluster algebras are always F-regular. This is joint work with Angelica Benito, Greg Muller and Jenna Rajchgot.

Shunsuke Tagaki (University of Tokyo, Japan)
Vanishing theorems on globally F-regular varieties

Globally F-regular varieties are a special type of Frobenius split varieties and closely related to log Fano varieties. Toric and Schubert varieties (in positive characteristic) are important examples of globally F-regular varieties. I will discuss vanishing theorems, and in particular Kollar's injectivity theorem for semi-ample line bundles, on globally F-regular varieties. This talk is based on joint work with Yoshinori Gongyo.

Kevin Tucker (University of Illinois - Chicago)
p_g Vector Bundles on Normal Surfaces

It is well-known that complete (or integrally closed) ideals on a rational surface singularities satisfy a number of special properties -- for example, a product of complete ideals is complete. While similar statements fail for general normal surfaces, they hold for a special class of complete ideals introduced by Okuma-Watanabe-Yoshida called p_g ideals. In this talk, I will discuss a higher-rank generalization of these concepts, called p_g vector bundles and some of their basic properties. This is joint work with Lawrence Ein.

Kei-ichi Watanabe (Nihon University, Japan)
Normal reduction numbers of integrally closed ideals in a 2-dimensional cone singularities.

This is a joint work in progress with Tomoyuki Okuma and Ken-ichi Yoshida.

Let (A, m, k) be an excellent normal 2-dimensional local ring and I be an integrally closed m-primary ideal. We assume k is algebraically closed and $k \subset A$.

For any resolution of singularity $f: X \to Spec(A)$, we define $p_g(A) = dim_k\, H^1(X, \mathcal{O}_X)$. A is a rational singularity if $p_g(A) = 0$.

For a minimal reduction Q of I, we calculate the numbers;

$\mathrm{nr}(I) = \min\{n \mid \overline{I^{n+1}} = Q\overline{I^n}\}$, $\quad \bar{r}(I) = \min\{n \mid \overline{I^{N+1}} = Q\overline{I^N}, \forall N \geq n\}$

and

$\mathrm{nr}(A) = \max\{\mathrm{nr}(I)\}$, $\bar{r}(A) = \max\{\bar{r}(I)\}$ $\quad (I \subset A, m-$ primary integrally closed ideals)

Example. $\bar{r}(A) = 1$ if and only A is a rational singularity and if A is an elliptic singularity, then $\bar{r}(A) = 2$. In general, we can show $\bar{r}(A) \leq p_g(A) + 1$.

We discuss $\bar{A}$ in case of cone singularities (we say A is a cone singularity if the exceptional set of the minimal resolution of A is a single smooth irreducible curve) using a vanishing theorem of Röhr. In particular, we will show;

Theorem. If A is a hypersurface singularity of dimension 2 defined by a homogeneous polynomial of degree $d \geq 4$, then $\bar{r}(A) = \mathrm{nr}(m) = d - 1$.

Also, we will show an example of A and I such that $\mathrm{nr}(I) = 1$ and $\bar{r}(I) = p_g(A) + 1$ for every $p_g(A) \geq 2$.