INTRODUCTION

The stereographic projection simplifies graphical solutions to problems involving the relative orientations of lines and planes in space. In rock mechanics contexts, stereographic projection is appealing for analyzing the stability of excavations as shown in Chapter 8, as well as for exploring and characterizing discontinuities in rocks. Many publications in structural geology, crystallography, and rock mechanics show constructions and tricks available using stereographic projection. Especially helpful references for the purposes of rock mechanics are Phillips (1972), Hoek and Bray (1977), and Goodman (1976). For the restricted applications considered in this volume, it will suffice to explain the underlying principles and to demonstrate the most essential operations.

Figure A5.1a shows the stereographic projection of a plunging line. The line passes through the center of a reference sphere at 0, and pierces its surface at $P$ in the lower hemisphere, and at $-P$ in the upper hemisphere. In all applications we will cling to the convention that the line or plane we wish to project contains the center of the reference sphere. The horizontal plane through 0 is termed the *projection plane*. A perpendicular to the projection plane pierces the top of the reference sphere at $F$, which will be termed the *focus for lower hemisphere projection*. The stereographic projection consists of projection of lines and points on the surface of a reference sphere from a single perspective point to corresponding points in the projection plane. To find the lower hemisphere stereographic projection of any line through 0 we find the point where this line pierces the surface of the reference sphere, construct a straight line between the piercing point and $F$, and
find the point where the construction line to $F$ crosses the projection plane. For example, the line $OP$ in Figure A5.1a pierces the reference sphere at point $P$ and the construction line $PF$ crosses the projection plane at point $p$. The latter is then the correct representation of $OP$ in a lower hemisphere stereographic projection. Similarly, the opposite end of $OP$, which pierces the upper hemisphere of the reference sphere at point $-P$, projects to point $-p$ as shown. Figure A5.1b presents a vertical section of the reference sphere through the line $0P$. It is perhaps easier to visualize the spatial relations of the line and its projection in this slice. Although the construction shown in Figure A5.1b, or its mathematical equivalent, can always be invoked to locate the stereographic projection of a line, it proves most convenient to plot the stereographic projection by tracing from a stereonet, as shown later.

The stereographic projection of a plane consists of finding the locus connecting the stereographic projections of all the lines it contains. A theorem holds that any circle on the reference sphere projects as a circle in the projection plane. (This is not true for the “equal area projection,” a variant of the stereographic projection.) Since any plane we wish to project must contain the center of the reference sphere, it must pierce the surface of the sphere along a great circle. In view of the above theorem, the stereographic projection of a plane must therefore project as a true circle. To find its center, it is sufficient to construct a circle through the stereographic projections of the strike line and the dip vector.

![Figure A5.2](image-url)
Figure A5.2 shows a horizontal plane piercing the reference sphere along great circle $SMT$. These points are unmoved by the projection from $F$. Therefore, a circle centered about $0$ in the projection plane represents the stereographic projection of a horizontal plane. Points inside it, when projected from $F$ at the top of the reference sphere, belong to the lower hemisphere; all other points belong to the upper hemisphere. This figure also shows an inclined plane passing through $0$ and intersecting the reference sphere along great circle $SdT$. Line $0S$ and its opposite $0T$ represent the strike of the inclined plane; these project at points $S$ and $T$. Line $0D$ is the dip vector of the inclined plane; it projects to point $d$ as shown. Other lines in the plane, $0A, 0B, 0C$, etc. project to points $a, b, c$, etc. to define the circular locus $Tdq$ as shown. To plot this locus is to determine the stereographic
PROJECTION OF A LINE

Line 1 plunges with vertical angle 40° below horizontal toward the N 30° E, plot on a lower hemisphere stereographic projection. The line will be assumed to lie atop the reference sphere, its stereographic projection will be a point inside the horizontal circle. The horizontal line passing through the point and N 30° E is then projected onto the stereographic projection. The point thus marked, point 1, is the stereographic projection we require.

A stereographic projection of a set of reference planes and lines are projected onto the lower hemisphere stereographic projection. Figure A5.3 is an equatorial stereographic presentation. The horizontal circle making a constant angle with the line of intersections. The small circles, which resemble the lines of latitude on a map of the globe, are the great circles of great circles. These great circles resemble the lines of longitude on a map of the globe, with the small circles making a constant angle with the line of intersections. The great circles, that is, any great circle are measured by counting the number of degrees. The point of intersection of the lines making a constant angle with the line of intersections is found.
THE ANGLE BETWEEN TWO LINES

A line (2) plunges 20° to the N 20° W. Plot this line and measure its angle with line 1 plotted previously. Line 2 is added to the tracing using the same sequence of steps as above (Figure A5.4c). To measure the angle from line 1 to line 2, it is now necessary to determine the plane common to both lines. Since each line passes through the center of the reference sphere, a common plane exists. It is found by rotating the tracing until both points fall along the same great circle (Figure A5.4d). The angle between (1) and (2) is then measured by counting the intersections with the small circles (which are spaced every 2°). The angle is 47°. The strike and dip of the plane common to (1) and (2) are indicated on Figure A5.4e, obtained by rotating the tracing so that the point of intersection of the great circle and the horizontal circle overlies the axis of the great circle family on the stereonet. Figure A5.4f shows the tracing at the end of this step.

PROJECTION OF A PLANE GIVEN ITS STRIKE AND DIP

Plot the stereographic projection of a plane (1) striking N 50° E and dipping 20° to the N 40° W. On a new tracing, the strike vector, a horizontal line bearing N 50° E, is plotted as a point 50° east of north along the horizontal circle (Figure A5.5a). Next rotate the tracing to place the strike vector over the axis of the great circles and plot the dip vector along the diameter at right angles to the strike (Figure A5.5b). The dip vector is a line plunging 20° to the N 40° W, so this step follows from the example discussed previously. Now trace the great circle common to the strike and dip vectors. To increase accuracy, the great circle may be constructed with a compass. Since the dip vector plunges 20°, the center of the great circle is 40° from vertical along the diameter containing the dip vector, as shown in Figure A5.5b.

Plot the stereographic projection of a plane (2) striking N 60° W and dipping 45° to the S 30° W and find the bearing and plunge of the line of intersection of planes 1 and 2. Emulating the steps above for plane 1, the new plane (2) yields the great circle shown in Figure A5.5c. This circle crosses the previously constructed great circle at the point marked \( I_{12} \). Since \( I_{12} \) is a point in the projection of each plane, it represents a line that lies in each plane; it is therefore the required intersection. The bearing and plunge of \( I_{12} \) are read from the stereonet by rotating the tracing to the diameter of the net as shown in Figure A5.5d. In this position, the vertical angle from \( I_{12} \) to horizontal (the plunge of \( I_{12} \)) can be found by counting the number of great circle intersections between \( I_{12} \) and horizontal. The intersection plunges 16° to the N 77° W. Figure A5.5e shows the tracing at the conclusion of this step.

There is a more convenient way to find the line of intersection of two planes when they are represented by their normals. If it is understood that a line identified as a
normal to a plane is intended to represent the plane, then in place of the great circle a plane can be plotted by means of a single point. To find the intersection line of two planes plotted by their normals, \( n_1 \) and \( n_2 \), the method shown in Figure A5.6 can be used. In this figure, the projections of planes 1 and 2 found in the previous example have been traced on a clean overlay. The normal to plane 1 (\( n_1 \)) is plotted in Figure A5.6a by lining up the dip vector of plane 1 with the diameter of the stereonet and measuring 90° along this diameter through the vertical. (A vertical line is represented by the point at the center of the projection, this being a lower hemisphere projection.) The normal to plane 2 is plotted similarly in Figure A5.6b. Then, in Figure A5.6c, the two normals, \( n_1 \) and \( n_2 \), are lined up on a common great circle by rotating the tracing appropriately. The normal to this great circle is \( I_{12} \) (Figure A5.6c). Figure A5.6d shows the tracing at the end of this step. Note that it was not necessary to draw the great circles of planes 1 and 2 in order to find \( I_{12} \) by this construction. They were drawn in the figure to demonstrate that the two methods of construction do in fact lead to the same result.
THE LOCUS OF LINES EQUIDISTANT FROM A GIVEN LINE

The locus of lines making a constant angle with a certain line is a circular cone with vertex at the center of the reference sphere. This cone projects as a small circle. By the theorem stated previously, the projection of a small circle is a true circle, that is, it may be drawn with a compass. A way to do this is shown in Figure A5.7.

Plot the locus of lines at 45° with the normal to plane 1 from the previous problem. In Figure A5.7a, the point \( n_1 \), traced from the Figure A5.6d, has been lined up with the net's diameter. Two lines on the cone are then plotted by moving away from \( n_1 \) by the required 45° along the diameter in each direction. In Figure A5.7b, the distance between these two points is then bisected to find the center of the small circle. Note that the center for construction does not coincide with the axis of the cone \( (n_1) \). The circle is drawn from the center using a compass as shown in Figure A5.7c. The tracing after this step is shown in Figure A5.7d.

VECTORS

Problems in slope stability and rock foundations involve manipulations with vectors. Since the direction of a vector can be shown as a point on the stereographic projection, the preceding constructions prove applicable to stability analysis, as discussed in Chapter 8. However, there is a world of difference between the tip and the tail of a vector in this connotation; therefore one must distinguish carefully between a line and its opposite. In structural geology work, such a distinction is not usually required and a point on one hemisphere can be replaced by its opposite in the other hemisphere without penalty. In rock mechanics we must work with the whole sphere.

There is no essential difficulty in working with both hemispheres, the only requirement being either a very large piece of paper or two separate tracings, one marked L.H. to denote lower hemisphere (as in all the examples of this section) and another marked U.H. to identify the upper hemisphere. Both hemispheres can be shown on one plot but only one hemisphere is located inside the horizontal circle. Constructions and manipulations helpful for problems embracing the whole sphere are presented by Goodman (1976).

REFERENCES


**PROBLEMS**

1. Write an equation for the distance from the center of the projection circle (corresponding to a unit reference sphere) to the lower hemisphere stereographic projection of a line plunging δ° below horizontal.

2. Determine the angle between lines 1 and 2 and the strike and dip of their common plane: Line 1 plunges 70° to the N 30° E. Line 2 plunges 15° to the N 60° E.

3. Determine the bearing of the plunge of the line of intersection of two planes given as follows: plane 1 strikes N 70° E and dips 60° S 20° E; plane 2 strikes N 20° W and dips 40° N 70° E.

4. Show how a line in the upper hemisphere is plotted in a lower hemisphere projection.

5. Given a line plunging in the lower hemisphere 30° to the north, plot its opposite in an upper hemisphere projection (i.e., a projection in which the focus is at the bottom of the reference sphere). Also, plot the line (not its opposite) in the lower hemisphere projection. Compare the two results and generalize if possible.

6. Construct the locus of lines making an angle of 35° with a line plunging 60° to the N 30° E. (Use a lower hemisphere projection.) What is the minimum angle between line 2 of Problem 2 and any point on the locus?