Design 1 Calculations

The following calculations are based on the method employed by Java Module A and are consistent with ACI318-99.

The values in Fig. 1 below were taken from the Design 1 Example found in the paper at the following web site: [http://www.nd.edu/~concrete/java/papers/manuscript-H0203-CAEE.pdf](http://www.nd.edu/~concrete/java/papers/manuscript-H0203-CAEE.pdf). The units have been changed to English in order to facilitate better understanding by students at the undergraduate level who generally deal with the English unit system.

<table>
<thead>
<tr>
<th>Session Input</th>
<th>Load Condition at Supports</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M_L: -221.24 k-ft</td>
</tr>
<tr>
<td></td>
<td>M_R: -221.24</td>
</tr>
<tr>
<td></td>
<td>w_L: 4.80 k/ft</td>
</tr>
<tr>
<td></td>
<td>w_R: 4.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No. Concentrated Loads</th>
<th>Load Magnitude</th>
<th>Load Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
</tr>
</tbody>
</table>

**Fig 1.** Session input for Design 1.

- **Calculation of Mu and Required Steel Area:**

  Assume that the beam is simply supported (in order to avoid static indeterminacy) and find the moment equation for both the max span condition (section 2) and the max support condition (sections 1 and 3) using statics. These equations have units of feet for x.

  1. \[ M_2 = -3.085x^2 + 75.92x - 147.49, \]

  2. \[ M_{1\&3} = -2.4x^2 + 59.064x - 221.24. \]

  Thus, the maximum moments, Mu, for each section are:

  \[ Mu_2 = 319.6 \text{ k-ft and } Mu_{1\&3} = 221.24 \text{ k-ft.} \]
Now find R in order to determine the steel ratio needed. The value of b used in this module is equal to:

(3) \[ b = \frac{h}{(h/b)} \]

rounded to the next highest whole number. The value for h used in this example, however, was kept at 23.62” because that number was converted from a dimension that was whole in centimeters. To be conservative, the value of d used in the primary calculation of R is:

(4) \[ d = h - cv - ds - \frac{sb}{2} - d_{\text{max}}, \]

where \( d_{\text{max}} \) is the maximum possible diameter (in this case 1.00”).

(5) \[ R = \frac{M_u}{(\Phi bd^2)}, \]

(6) \[ \rho = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \]

where \( a = -0.59(f_y^2)/(f_c'), b = f_y, \) and \( c = -R. \)

This can be used to calculate the required steel area from:

(7) \[ A_{\text{req}} = \rho bd. \]

See Fig. 2 below for a summary of these calculations.

<table>
<thead>
<tr>
<th>Section</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (ft)</td>
<td>24.61</td>
<td>24.61</td>
<td>24.61</td>
</tr>
<tr>
<td>( M_u ) (k-ft)</td>
<td>221.24</td>
<td>319.60</td>
<td>221.24</td>
</tr>
<tr>
<td>R (lb/ in²)</td>
<td>522.43</td>
<td>754.70</td>
<td>522.43</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.009838</td>
<td>0.014923</td>
<td>0.009838</td>
</tr>
<tr>
<td>( A_{\text{req}} ) (in²)</td>
<td>2.766</td>
<td>4.196</td>
<td>2.766</td>
</tr>
</tbody>
</table>

Fig 2. Summary of moment and \( A_{\text{req}} \) calculations.

**Combinations of Steel**

Module A first determines the single bar combinations for the required steel area using the minimum and maximum bar size numbers (in this case No. 6 & 8). The calculations for this can be seen in Fig. 3.

The underlined values correspond to areas that exceed the \( A_{\text{req}} \) for the maximum support moment and the bolded values correspond to the maximum span moment. Once these single bar combinations have been determined, they are checked to ensure that the clear spacing between bars is suitable (the user defines this clear spacing to be \( sb \) in the Module).
The lowest area for both the max support and max span are then used to find combinations of two different bar numbers that have a smaller area but a large enough capacity. This decision, however, also depends on symmetry. The acceptable and unacceptable steel configurations can be seen in Fig. 4.

<table>
<thead>
<tr>
<th># of bars</th>
<th>#6</th>
<th>#7</th>
<th>#8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.442</td>
<td>0.601</td>
<td>0.785</td>
</tr>
<tr>
<td>2</td>
<td>0.884</td>
<td>1.203</td>
<td>1.571</td>
</tr>
<tr>
<td>3</td>
<td>1.325</td>
<td>1.804</td>
<td>2.356</td>
</tr>
<tr>
<td>4</td>
<td>1.767</td>
<td>2.405</td>
<td>3.142</td>
</tr>
<tr>
<td>5</td>
<td>2.209</td>
<td>3.007</td>
<td>3.927</td>
</tr>
<tr>
<td>6</td>
<td>2.651</td>
<td>3.608</td>
<td>4.712</td>
</tr>
<tr>
<td>7</td>
<td>3.093</td>
<td><strong>4.209</strong></td>
<td>5.498</td>
</tr>
<tr>
<td>8</td>
<td>3.534</td>
<td>4.811</td>
<td>6.283</td>
</tr>
<tr>
<td>9</td>
<td>3.976</td>
<td>5.412</td>
<td>7.069</td>
</tr>
<tr>
<td>10</td>
<td><strong>4.418</strong></td>
<td>6.013</td>
<td>7.854</td>
</tr>
<tr>
<td>11</td>
<td>4.860</td>
<td>6.615</td>
<td>8.639</td>
</tr>
<tr>
<td>12</td>
<td>5.301</td>
<td>7.216</td>
<td>9.425</td>
</tr>
</tbody>
</table>

**Fig 3.** Single bar combinations for Design 1.

In order to facilitate an easier calculation of the possible bar combinations, all of the combinations of steel were calculated and then it was determined which ones were symmetric (and thus acceptable arrangements). The tables in Fig. 5 show these calculations. The steel areas were carried to three decimal places because the module does not use the bar areas found in most textbook tables. Values are not truncated not rounded in the module. The shaded cells indicate that the combination is symmetric; thus, these are the only ones considered in the steel choice.

**Fig 4.** Acceptable and unacceptable bar configurations.
### # 7's

<table>
<thead>
<tr>
<th>db=</th>
<th># 6's</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>Most in 1 Row= 5</td>
<td>0.875</td>
<td>0.000</td>
<td>0.601</td>
<td>1.203</td>
<td>1.804</td>
<td>2.405</td>
<td>3.007</td>
</tr>
<tr>
<td>1</td>
<td>0.442</td>
<td>1.043</td>
<td>1.644</td>
<td>2.246</td>
<td>2.847</td>
<td>3.448</td>
<td>4.050</td>
<td></td>
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<tr>
<td>2</td>
<td>0.884</td>
<td>1.485</td>
<td>2.086</td>
<td>2.688</td>
<td>3.289</td>
<td>3.890</td>
<td>4.491</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.325</td>
<td>1.927</td>
<td>2.528</td>
<td>3.129</td>
<td>3.731</td>
<td>4.332</td>
<td>4.933</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.767</td>
<td>2.368</td>
<td>2.970</td>
<td>3.571</td>
<td>4.172</td>
<td>4.774</td>
<td>5.375</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.209</td>
<td>2.810</td>
<td>3.412</td>
<td>4.013</td>
<td>4.614</td>
<td>5.216</td>
<td>5.817</td>
<td></td>
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<tr>
<td>6</td>
<td>2.651</td>
<td>3.252</td>
<td>3.853</td>
<td>4.455</td>
<td>5.056</td>
<td>5.657</td>
<td>6.259</td>
<td></td>
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<tr>
<td>8</td>
<td>3.534</td>
<td>4.136</td>
<td>4.737</td>
<td>5.338</td>
<td>5.940</td>
<td>6.541</td>
<td>7.142</td>
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</tr>
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### # 8's

<table>
<thead>
<tr>
<th>db=</th>
<th># 6's</th>
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<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>1.00</td>
<td>Most in 1 Row= 4</td>
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<td>0.000</td>
<td>0.785</td>
<td>1.571</td>
</tr>
<tr>
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<td>2.013</td>
<td>2.798</td>
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<tr>
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<td>1.669</td>
<td>2.454</td>
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<tr>
<td>3</td>
<td>1.325</td>
<td>2.111</td>
<td>2.896</td>
<td>3.682</td>
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<td>2.553</td>
<td>3.338</td>
<td>4.123</td>
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<td>3.780</td>
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<td>4.320</td>
<td>5.105</td>
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</tbody>
</table>

### # 8's

<table>
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<th># 7's</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>Most in 1 Row= 4</td>
<td>1.00</td>
<td>0.000</td>
<td>0.785</td>
<td>1.571</td>
</tr>
<tr>
<td>1</td>
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<tr>
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<td>1.203</td>
<td>1.988</td>
<td>2.773</td>
<td>3.559</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.804</td>
<td>2.589</td>
<td>3.375</td>
<td>4.160</td>
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<tr>
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<td>2.405</td>
<td>3.191</td>
<td>3.976</td>
<td>4.761</td>
<td></td>
</tr>
<tr>
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<td>3.007</td>
<td>3.792</td>
<td>4.577</td>
<td>5.363</td>
<td></td>
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<tr>
<td>6</td>
<td>3.608</td>
<td>4.393</td>
<td>5.179</td>
<td>5.964</td>
<td></td>
</tr>
</tbody>
</table>

**Fig 5.** Steel combinations for two different bar sizes. Shaded cells indicate symmetric (acceptable) combinations. Underlined cells are combinations that provide a steel area less than the single bar combination for the max support moment.

- **Steel Selection, Spacing Between Bars, and Moment Capacity**

**Max Span Moment Steel:** For this example (Design 1), there were no steel combinations using two different bar sizes that provided an area less than that of the 7, #7 combination for the max span moment. Thus, for Section 2 use 2 layers with 5, #7 bars in the lower layer.
Max Support Moment Steel: There are three combinations which need to be checked for Sections 1 & 3. The calculations can be seen in Fig. 6. The combination that is chosen is 2, #8 and 3, #6 because the in design it is better to choose bar numbers that are not consecutive bar numbers.

<table>
<thead>
<tr>
<th>Bars 6 &amp; 7 Combo</th>
<th>Asprov (in²)</th>
<th>a</th>
<th>d one row</th>
<th>Capacity</th>
<th>Spacing One Row</th>
<th>Most in Two Rows d</th>
<th>Capacity</th>
<th>Spacing Two Rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,#7;5,#6</td>
<td>2.96</td>
<td>3.618</td>
<td>21.24</td>
<td>0.63</td>
<td>244.4</td>
<td>1.22</td>
<td>20.33</td>
<td>238.5</td>
</tr>
<tr>
<td>2,#8;3,#6</td>
<td>2.90</td>
<td>3.540</td>
<td>21.17</td>
<td>1.71</td>
<td>250.1</td>
<td>2.04</td>
<td>0.63</td>
<td>221.24</td>
</tr>
<tr>
<td>2,#8;2,#7</td>
<td>2.77</td>
<td>3.390</td>
<td>21.17</td>
<td>235.0</td>
<td>1.71</td>
<td></td>
<td>0.63</td>
<td>235.0</td>
</tr>
</tbody>
</table>

**Fig 6.** Maximum support reinforcement calculations.

The value of d is determined from Eq. (4), but instead of using d_max, the actual d is used for the larger bar diameter (in this case it is still 1.00”). The moment capacity is determined from:

(8) \( Mn = 0.9A_s f_y (d-a/2) \),

where a is the rectangular stress block depth found from:

(9) \( a = A_s f_y / (0.85 f_{cyb}) \).

A summary of the selected steel and their moment capacities can be seen in Fig. 7.
Cut-off Locations

Positive Reinforcement
The cut-off location for the positive (bottom) reinforcement is the larger of the development length, $l_d$, and the theoretical cut-off point plus $d$ or $12\text{db}$ (whichever is larger). Since the user input, $rb$ ($\%$), is 66.7, cut off 4 out of the 7 #7 bars. This leaves 1.80 in$^2$.

Development Length
The development length is found from:

$$l_d = \frac{f_y \alpha \beta \lambda}{(k \sqrt{f_{c}^{-1}})},$$

where $\alpha = 1.0$ (bottom bars), $\beta = 1.0$ (uncoated), and $\lambda = 1.0$ (normal weight concrete). The constant $k$ is dependent on the size of bar used and clear cover (see ACI318-99). In this example, $k = 20$. Thus,

$$l_d = 58(0.875)/((20)(\sqrt{3987.5})) = 40.2''$$

from the midspan.

Using a reduction factor of $\frac{A_{req}}{A_{prov}} = 4.19/4.21$, $l_d = 39.9''$.

Theoretical Cut-off Point + $d$ or $12\text{db}$
The theoretical cut-off point is found from determining where the remaining bars will have enough capacity to support the demand. This occurs when Eq. (1) is equal to Eq. (8) for an $A_{\text{remaining}} = 1.80$ in$^2$.

$$a = 1.8(58)/((0.85)(3.9875)(14)) = 2.20 \text{ in.}$$

$$d = 23.62 - 1.57 - 0.374 - 0.5(7/8) = 21.24 \text{ in.}$$

$$M_n = 0.9(1.8)(58)(21.24 - 2.2/2) = 157.7 \text{ k-ft}$$

Eq. (1) = $M_n$ when $x = 60.7$ in. from the support. Comparing $d$ and $12\text{db}$ ($10.5''$), $d$ governs and therefore the length is

$$60.7'' - 21.24'' = 39.5''$$

from the support.

Thus, the governing cut-off length for the positive reinforcement is 39.5” from the support. The remaining 3, #7 are assumed to be continuous all the way into the support.

Negative Reinforcement
The cut-off location for 60% of the negative (top) reinforcement ($rt$ ($\%$)) is the larger of the development length, $l_d$, and the theoretical cut-off point plus $d$, $12\text{db}$, or $1/16L$ (whichever is largest). Since the user input, $rt$ ($\%$), is 60, cut off 3, #6 bars and leave 2, #8’s. This leaves 1.57 in$^2$. 
Development Length
The development length is:

\[ l_d = 58(0.75)/((25)(\sqrt{3987.5})) = 27.55" \text{ from the support.} \]

Using a reduction factor of \( \frac{A_{\text{req}}}{A_{\text{prov}}} = 2.765/2.896 \), \( l_d = 26.3" \).

Theoretical Cut-off Point + d, 12db, or 1/16L

\[ A_{\text{remaining}} = 1.57 \text{ in}^2. \]

\[ a = 1.57(58)/((0.85)(3.9875)(14)) = 1.92 \text{ in.} \]
\[ d = 23.62 - 1.57 - 0.374 - 0.5(8/8) = 21.176 \text{ in.} \]
\[ Mn = 0.9(1.57)(58)(21.176 - 1.92/2) = 138.07 \text{ k-ft} \]

Eq. (1) = Mn when \( x = 18 \text{ in.} \) from the support. Comparing \( d, 12\text{db} \) (10.5"), and 1/16L (18.5"), \( d \) governs and therefore the length is

\[ 18" + 21.176" = 39.2" \text{ from the support.} \]

Thus, the governing cut-off length for the 3, #6 in the top reinforcement is 39.2" from the support.

The cut-off location for the remaining 2, #8 bars is equal to the larger of \( l_d \) from the theoretical cut-off point and the inflection point on the moment diagram + d, 12db, or 1/16L.

Development Length
The development length is:

\[ l_d = 58(1)/((20)(\sqrt{3987.5})) = 45.9" \text{ from the support.} \]

Using a reduction factor of \( \frac{A_{\text{req}}}{A_{\text{prov}}} = 2.765/2.896 \), \( l_d = 43.84" \).

From the theoretical cut-off point (18" from the support), \( x = 61.8" \).

Thus, the governing cut-off length for the 3, #8 in the top reinforcement is 76.5" from the support.

Inflection Point
The inflection point is where the moment is zero on the demand curve from the negative side. From graphing, this occurs at \( x = 55.3" \). Adding on \( d \) to this because it governs results in a cut-off point = 76.5".

Thus, the governing cut-off length for the 3, #8 in the top reinforcement is 76.5" from the support.
Module Display Windows

Fig 8. Design details at Section 1 (the left support).

Fig 9. Cut-off locations for positive and negative reinforcement.
Fig 10. Design flexural capacity and factored design flexural demand along the length of the beam.

Fig 11. Typical properties displayed in Window 4 of Module A. Left: Max. positive moment vs. bottom steel area. Right: Max. left end negative moment vs. top steel area.