Updated 7 August, 2015

CONFERENCE ON LIE ALGEBRAS, VERTEX OPERATOR ALGEBRAS, AND RELATED TOPICS

A conference in honor of J. Lepowsky and R. Wilson
Department of Mathematics – University of Notre Dame
Friday August 14 – Tuesday August 18, 2015

TITLES AND ABSTRACTS

Drazen Adamovic, University of Zagreb, Croatia
Explicit realizations of affine vertex algebras and their applications

We shall present explicit realization of certain affine and superconformal vertex algebras and their modules. These realizations show that admissible affine vertex algebras are closely related with W-algebras appearing in logarithmic conformal field theory. We shall discuss relations with triplet vertex algebras and their generalizations (studied in a joint project with A. Milas). We shall also present a solution of the irreducibility problem for the Wakimoto modules for the affine Lie algebra $\widehat{sl_2}$ and their generalizations. A principal realization of the Wakimoto modules at the critical level (obtained in a joint work with N. Jing and K. Misra) will be discussed. A combinatorial basis of such modules will be constructed.

Darlayne Addabbo, University of Illinois at Urbana–Champaign
Representation theory and generalized Q-systems

We will discuss certain tau functions for the Toda lattice, given as matrix elements for the homogeneous realization of the basic representation of $\widehat{sl_2}$. These tau functions satisfy difference relations called a Q-system. We will discuss analogous tau functions given as matrix elements for the basic representation of $\widehat{sl_3}$ and the progress we have made in understanding what sort of system they comprise.

George Andrews, Pennsylvania State University
On the Kanade/Russell Conjectures

At the Joint Mathematics Meetings in January, 2015, Jim Lepowsky discussed several tantalizing conjectures of S. Kanade and M. Russell (two graduate students at Rutgers). Three of these conjectures fall well outside of the classical results in the theory of partitions connected with modular forms (e.g. the Rogers-Ramanujan Identities). In this talk, I will discuss the history of problems of this nature. I will describe the known methods of proof, and I will discuss the application of these methods to the Kanade/Russell problems.

Georgia Benkart, University of Wisconsin
A walk on the McKay correspondence

The McKay Correspondence establishes amazing connections between the finite subgroups of SU(2) and the simply-laced affine Dynkin diagrams. Walks on the Dynkin diagrams correspond to tensor powers of the natural two-dimensional SU(2)-module, and hence in the exceptional cases, with tensor powers of the spin module for the double cover of the symmetric and alternating groups S_4, A_4, A_5. This talk will explore some of the combinatorial connections that such walks have with partitions and set partitions.

Kathrin Bringmann, University of Cologne, Germany
Quantum modular forms and application

In my talk I want to discuss a new class of functions which has been introduced by Don Zagier, namely quantum modular forms. These functions, which resemble modular forms on the real axis, have recently obtained a lot of attention. I want to in particular explain how they relate to the mock theta functions of Ramanujan and naturally occur in various areas including W-algebras.

Vyjayanthi Chari, University of California – Riverside
Multiplicities in Demazure flags, mock theta functions and the cone theta functions

We discuss the Demazure modules occurring in level one representations of affine Lie algebras. It was shown by Naoi, that one can use results of Joseph and Lusztig, to prove that (under certain conditions) these modules admit a flag by Demazure modules occurring in level m representations of affine Lie algebras. We shall see that there is an alternate and more constructive way to prove this result. This allows us to compute the multiplicity of a level m Demazure module occurring in a flag of a level one Demazure module. In the case of the affine Lie algebra associated to $\mathfrak{sl}_2$, we shall see that the generating function which encodes the multiplicity has a specialization which is related to the fifth order mock theta function of Ramanujan. In the case of $\mathfrak{sl}_n$, we shall see that the level two flag is related to the cone theta function. The talk is based on a paper with Biswal, Schneider, Viswanath and work in progress with Biswal, Shereen and Wand.

Corina Calinescu, The City University of New York
On vertex-algebraic structure of principal subspaces of standard modules for affine Lie algebras

The theory of principal subspaces of standard modules for untwisted affine Lie algebras was initiated by B. Feigin and A. Stoyanovsky, and has been further developed by several authors from different standpoints. This talk is an overview of work done jointly with J. Lepowsky and A. Milas on vertex-algebraic structure of principal subspaces of standard modules for certain untwisted and twisted affine Lie algebras. I will discuss the algebraic

and combinatorial properties of these subspaces, by using mainly the theory of vertex operator algebras and intertwining operators.

Lisa Carbone, Rutgers University
Finite presentations of hyperbolic Kac-Moody groups

Tits defined Kac–Moody groups over commutative rings, providing infinite dimensional analogues of the Chevalley-Demazure group schemes. Tits' presentation can be simplified considerably when the Dynkin diagram is hyperbolic and simply laced. Over finitely generated rings R, we give finitely many generators and defining relations parametrized over R and we describe a further simplification for $R = \mathbb{Z}$. We highlight the role of the group $E_{10}(R)$, conjectured to play a role in the unification of superstring theories.

Bud Coulson, Rutgers University
An $A_2^{(1)}$ motivated proof of Rogers-Ramanujan identities

I will present a new proof of the Rogers-Ramanujan identities, based off of the denominator identity for the affine Lie algebra $A_2^{(1)}$. This is part of a program of so-called "motivated proofs" for identities of the Rogers-Ramanujan type.

Chongying Dong, University of California – Santa Cruz
Parafermion vertex operator algebras

The representation theory for parafermion vertex algebra associated to any finite dimensional simple Lie algebra and any positive integer will be reported.

Jurgen Fuchs, Karlstad University, Sweden
Mapping class group invariants from finite ribbon categories

Given a factorizable finite ribbon category D, one can associate to any punctured surface M, with punctures labeled by objects of D, a finite-dimensional vector space $Bl(M)$. These spaces carry representations of mapping class groups and are compatible with sewing of surfaces. Thus, albeit D is not required to be semisimple, they behave like the spaces of conformal blocks of a rational conformal field theory. When restricting to surfaces M for which each puncture is labeled by one and the same object F of D, there is a natural construction that selects a specific vector $v(M, F)$ in each space $Bl(M)$. These vectors are invariant under the mapping class group action and are mapped to each other under sewing, if and only if the chosen object F carries a structure of an "S-invariant" commutative cocommutative symmetric Frobenius algebra in D. In particular, if there is a chiral conformal field theory based on a category which has D as its enveloping category, then F is the bulk state space of a full conformal field theory and the vectors $v(M, F)$ are the bulk correlation functions of that theory. (Joint with C. Schweigert.)

Andre Henriques, Utrecht University, Netherlands
Representations of based loop groups

Representations of free loop groups possess an operation, akin to tensor product, under which they form a braided tensor category. In VOA language, these are the representations of affine vertex algebras at positive integral level. I will discuss a similar operation, which is present on the category of representations of the based loop groups, and which equips it with the structure of a monoidal cateogory. Finally, I will present a recent result, according to which the Drinfel'd centre of the category of representations of a based loop group is equivalent to the category of representations of the corresponding free loop group.

Naihuan Jing, North Carolina State University
Quantum Sugawara operators of quantum affine algebra of type A

I will talk about the recent explicit formulae of quantum Sugawara operators in type A (jointly with Frappat, Molev and Ragoucy). The images of these elements under a Harish-Chandra-type homomorphism will also be given. These images reproduce generators of the q-deformed classical W-algebra of Frenkel and Reshetikhin.

Shashank Kanade, University of Alberta
The Rogers-Ramanujan identities: From sums, hopefully to products

Feigin-Stoyanovsky's "principal subspaces" of level 1 standard modules for $A_1^{(1)}$ exhibit the Rogers-Ramanujan "sum sides," i.e, the "difference-2" conditions as elegantly exhibited by Capparelli-Lepowsky-Milas and Calinescu-Lepowsky-Milas using vertex-operator-algebraic methods. In this talk, we will discuss some ideas and results for using vertex-operator-theoretic techniques to interpret the corresponding "product sides." To this end, stimulated by an idea of J. Lepowsky, we present some results on a certain Koszul complex related to the principal subspaces. The underlying constructions are surprisingly ubiquitous and arise in various settings, for instance, in Gorsky-Oblomkov-Rasmussen's analysis of the Khovanov homology of torus knots. Further work on this problem is in progress with J. Lepowsky and S. Sahi.

Yasuyuki Kawahigashi , University of Tokyo, Japan
From vertex operator algebras to operator algebras and back

Theory of vertex operator algebras and theory of local conformal nets based on operator algebras are two mathematical approaches to chiral conformal field theory, and it is clear that the two theories are similar in many aspects. We provide a direct relation between the two theories for the first time. That is, we start with a vertex operator algebra with so-called "strong locality" and then construct a local conformal net and also go back by recovering the original vertex operator algebra. We give a simple sufficient condition for

strong locality. This is a joint work with Carpi, Longo and Weiner.

Matthew Krauel, University of Cologne
A transformation law for vertex operator algebra theta functions

We consider a type of theta function associated to vertex operator algebras. These are a generalization of functions originally considered by Miyamoto. We discuss the problems an insertion in such functions can pose, and provide transformation properties in this setting.

Jim Lepowsky, Rutgers University
Representation theory of vertex operator algebras and partition identities – some current developments

I will sketch some recent and ongoing developments stimulated by Robert Wilson's and my collaboration relating vertex operator (algebra) theory to generalized Rogers-Ramanujan identities.

Andrew Linshaw, University of Denver
Cosets of affine vertex algebras inside larger structures

The commutant or coset construction is a standard way to construct new vertex algebras from old ones. Given a vertex algebra V and a subalgebra A of V, $\mathrm{Com}(A, V)$ is the subalgebra of V which commutes with A. It is widely believed that $\mathrm{Com}(A, V)$ will inherit nice properties of A and V such as rationality, C_2-cofiniteness, or strong finite generation, but no general results of this kind are known. In this talk, I will discuss my recent joint work with T. Creutzig (University of Alberta) in which we established the strong finite generation of a broad class of cosets of affine vertex algebras inside larger structures.

Masahiko Miyamoto, University of Tsukuba, Japan
The regularity of orbifold model

We show that if T is a simple regular VOA of CFT-type with a nonsingular invariant bilinear form and σ is a finite automorphism of T, then the fixed point subVOA T^σ is also regular.

Robert McRae, Beijing University
On the associativity isomorphisms in affine Lie algebra tensor category

For a simple Lie algebra $\mathfrak{g}$, Huang and Lepowsky showed that the category of standard modules for the affine Lie algebra $\hat{\mathfrak{g}}$ at a fixed non-negative integral level is a braided tensor category. This category of $\hat{\mathfrak{g}}$-modules is the category of modules for a certain simple vertex operator algebra, and Frenkel and Zhu showed that this category is equivalent to the category of finite-dimensional modules for a certain quotient of $U(\mathfrak{g})$. In particular, the associativity isomorphisms constructed by Huang and Lepowsky induce associativity isomorphisms in a certain category of finite-dimensional $\mathfrak{g}$-modules. In this talk, I will discuss

an explicit construction of these isomorphisms: they turn out to be related to "Drinfeld associators" constructed from the KZ equations.

Evgeny Mukhin, Indiana University – Purdue University Indianapolis
Integrable system associated to quantum toroidal $\mathfrak{gl}(1)$

We discuss a new approach to Bethe ansatz mathod for quantum integrable models associated to R-matrices of quantum groups. Our approach is based on a realization of algebras and representations via spaces of correlation functions and a description of the Hamiltonians as projections of simple multiplication operators. The main example for this talk is the quantum toroidal $\mathfrak{gl}(1)$ where no alternative approach is known. In this case, the Hamiltonians are closely related to integrals of motions generated by the deformed Virasoro algebra. This talk is based on a joint project with B. Feigin, M. Jimbo, and T. Miwa.

Kiyokazu Nagatomo, Osaka University, Japan
Vertex operator algebras with central charge 1/2 and -68/7

In this talk we study rational C_2-cofinite simple vertex operator algebras whose spaces of characters are 3-dimensional, and satisfy 3rd order (modular) linear differential equations. We classify such vertex operator algebras with central charge 1/2 and -68/7. One of the main results is that these vertex operator algebras have conformal weights $\{0, 1/2, 1/16\}$ and $\{0, -2/7, -3/7\}$, respectively, and are isomorphic to minimal series of Virasoro vertex operator algebras of central charge $c = c_{3,4} = 1/2$ and $c_{2,7} = -68/7$. We also refer to a classification of lattice vertex operator with central charge 8 and 16. (Joint work with Y. Sakai.)

Debajyoti Nandi, Chennai Mathematical Institute, India
Partition identities arising from the level 4 standard modules for $A_2^{(2)}$

I will present a new set of (conjectured) partition identities arising from a twisted vertex operator construction of "tight" spanning sets of the level 4 standard modules for the affine Lie algebra $A_2^{(2)}$.

Michael Penn, Colorado College
Vertex-algebraic structure of principal subspaces of standard $A_{2n}^{(2)}$*-modules*

The principal subspaces of standard modules for untwisted affine Lie algebras, introduced by Feigin and Stoyanovsky, has been an active area of study in recent years. One current focus in this area is the study of standard modules for twisted affine Lie algebras. In this work, we use vertex algebraic techniques to describe the principal subspaces of standard modules for the twisted affine Lie algebra $A_{2n}^{(2)}$. This is joint work with C. Calinescu and A. Milas.

Ozren Perse, University of Zagreb, Croatia
Representations of some affine vertex algebras at negative integer levels

In this talk we present a construction of singular vectors in some affine vertex operator algebras at negative integer levels. We use these results to obtain the branching rules for certain conformal embeddings. Part of the talk is based on join work with D. Adamovic.

Alexander Premet, University of Manchester, United Kingdom
Maximal subalgebras of exceptional Lie algebras over fields of good characteristic

Maximal connected subgroups of exceptional simple algebraic groups in positive characteristic have been classified in a series of paper by Seitz, Testerman and Liebeck–Seitz. In my talk, which is partly based on joint work with David Stewart, I am going to discuss the classification problem of maximal Lie subalgebras of Lie algebras of exceptional algebraic groups.

Mirko Primc, University of Zagreb, Croatia
Difference conditions in combinatorial bases of representations of affine Lie algebras

In this talk I will review some known constructions of combinatorial bases of standard level k modules of affine Lie algebras which are based on a defining relation $x\theta(z)k+1=0$, and describe a new construction for basic modules of affine Lie algebras of type C_n. I will also comment on a type of combinatorial difference conditions and the corresponding Rogers-Ramanujan type identities one should expect in general for higher rank algebras.

David Radnell, Aalto University, Finland
Analytic foundations of conformal field theory

The algebraic structures in conformal field theory are well understood thanks to the development of vertex operator algebras and their representation theory. The analytic structures have not been studied in the same depth. In particular, for the construction of high-genus conformal field theory, deep results in the complex geometry of the moduli space of Riemann surfaces with parametrized boundaries are required. Even the analytic setting for the definition of conformal theory is not completely settled. I will discuss the program to address these issues, recent results and future goals.

David Ridout, Australian National University, Australia
Non-C_2-cofinite VOAs and the Verlinde formula

While the C_2-cofinite triplet VOAs have received a lot of attention over the last ten years, there is strong physical motivation to also study non-C_2-cofinite VOAs. Recently, it has been shown that the most tractable examples admit a formalism that seems to generalise the paradigm of modularity and the Verlinde formula, familiar from rational VOAs. This talk will introduce and review this formalism, with the aid of an example, and explain how

it may resolve the contentious issue of a Verlinde formula for the triplet VOAs.

Ingo Runkel, University of Hamburg, Germany
Holomorphic symplectic fermions

From the conformal field theory of n pairs of symplectic fermions one obtains a non-semisimple braided tensor category $SF(n)$, which conjecturally agrees with the representations of the even part of the symplectic fermion VOA of central charge $-2n$. For n divisible by 8, $SF(n)$ contains a Lagrangian algebra, that is, a commutative algebra A, all of whose local modules are direct sums of A. Such algebras are the categorical counterpart of holomorphic extensions of a VOA. We argue that the corresponding extension of the symplectic fermion VOA is a lattice VOA for an n-dimensional even self-dual lattice with a deformed stress tensor. This is joint work with Alexei Davydov.

Matthew Russell, Rutgers University
IdentityFinder and some new identities of Rogers-Ramanujan type

The Rogers-Ramanujan identities and various analogous identities (Gordon, Andrews-Bressoud, Capparelli, etc.) form a family of very deep identities concerned with integer partitions. These identities (written in generating function form) are typically of the form "product side" equals "sum side," with the product side enumerating partitions obeying certain congruence conditions and the sum side obeying certain initial conditions and difference conditions (along with possibly other restrictions). We use symbolic computation to generate various such sum sides and then use Euler's algorithm to see which of them actually do produce elegant conjectured product sides. We not only rediscover many of the known identities but also discover some new ones, as conjectures supported by strong mathematical evidence. Joint work with Shashank Kanade.

Christopher Sadowski, Ursinus College
Vertex-algebraic structure of principal subspaces of $D_4^{(3)}$-modules

Principal subspaces of standard modules for untwisted affine Lie algebras were introduced by Feigin and Stoyanovsky, and have been further studied by many other authors. In this talk, we introduce and discuss principal subspaces of certain standard modules for the twisted affine Lie algebra $D_4^{(3)}$. We extend earlier results and vertex-algebraic techniques of Calinescu, Lepowsky, and Milas on principal subspaces of standard $A_2^{(2)}$-modules.

Alexei Semikhatov, Lebedev Physica Institute, Russia
The different facets of logarithmic Kazhdan–Lusztig duality

I discuss current understanding of "dualities" between Hopf algebras, logarithmic CFT models, and spin chains.

Stephan Stolz, University of Notre Dame

Factorization algebras, vertex operator algebras and functorial field theories

There are various, quite different mathematical approaches to quantum field theory, among them vertex operator algebras, functorial field theories in the sense of Atiyah and Segal, and the factorization algebras of quantum observables constructed by Costello and Gwilliam. We will outline the relationship between factorization algebras and vertex operator algebras (this is work of Costello/Gwilliam) and between factorization algebras and functorial field theories (joint work of Dwyer/Stolz/Teichner).

Nathan Vander Werf, University of Notre Dame

Special pairs of screening operators and certain subalgebras of lattice vertex operator algebras

We discuss the notion of special pairs of screening operators for a rank d lattice vertex operator superalgebra. Such screening pairs $(\tilde{Q}, Q)$ in the rank 1 case proved to be useful machinery in studying the internal structure of the $\mathcal{W}$-algebra $\mathcal{W}(p) = \ker \tilde{Q}$ and proving the C_2-cofinite property. From an analysis of screening pairs that can arise in the rank d case of a lattice L, one can construct a large number of subalgebras of V_L by considering the intersection of the kernels of certain screening operators that share a common Virasoro element. Some subalgebras that emerge share features similar to the $\mathcal{W}(p)$-algebra in the rank 1 setting while others do not as much. If time permits, we will show in the rank 2 setting how the kernel of a long screening operator $\ker \tilde{Q}$ (and in some cases the intersection of the kernels of two screening operators) can be computed. With such decompositions, the problem of studying the internal structure of $\ker \tilde{Q}$ such as finding a strongly generating set for $\ker \tilde{Q}$ and proving the C_2-cofinite property become tractable.

Nolan Wallach, University of California – San Diego

Structural results for Vinberg pairs

In this talk I will first recall Vinberg's theory of theta Groups (which are the Vinberg pairs of the title). In particular I will sketch a proof of Vinberg's main theorem without a case by case consideration of Kac's classification of automorphisms of finite order on simple Lie algebras over the complexes. The rest of the lecture will give a generalization of the Kostant-Rallis theorem to a large class of Vinberg pairs and describe several examples of these pairs that have had applications to physics.

Weiqiang Wang, University of Virginia

Super Kazhdan-Lusztig theory ABC

Kazhdan-Lusztig (KL) theory for semisimple Lie algebras has been well developed since 1979. For Lie superalgebras, a suitable KL theory is emerging more recently. We will explain our approach (joint with Cheng, Lam, and Bao) to a super KL theory for classical

Lie superalgebras (with emphasis on type A), using canonical bases.

Katrin Wendland, University of Freiburg, Germany
Towards vertex operator algebras for K3

The definition of vertex operator algebras and the known ingenious techniques for them, like certain orbifoldings, a priori restrict attention to cases which in physics are known as holomorphic conformal field theories. On the other hand, physics predicts a rich variety of applications of conformal field theory in geometry, where in general the restriction to holomorphic data has to be dropped.

In this talk, we review some aspects of conformal field theories that are associated to K3 surfaces. In particular, we present recent progress in understanding some of the underlying vertex operator algebras.

Evan Wilson, Rutgers University
Tensor product decomposition $\widehat{\mathfrak{sl}_n}$-modules and generating series

In this talk, we describe recent joint work with Kailash Misra on decomposing the tensor product of two level one modules of the affine Kac-Moody algebra $\widehat{\mathfrak{sl}_n}$, using the crystal basis of quantum $U(\widehat{\mathfrak{sl}_n})$ of Misra and Miwa and some well-known graded dimension formulas. In the process, we uncover some generating series for partitions whose parts satisfy certain conditions.

Simon Wood, The Australian National University, Australia
Symmetric polynomials and modules over affine $\mathfrak{sl}_2$ at admissible levels

At non-critical levels affine $\mathfrak{sl}_2$ can be used to construct a vertex operator algebra which contains a non-trivial ideal precisely when the level is admissible. In this talk I will explain how symmetric polynomials yield simple formulae for the singular vectors that generate these ideals and how these singular vectors with their associated symmetric polynomial data can be used to easily classify modules over the simple quotient vertex operator algebra.

Jinwei Yang, University of Notre Dame
Vertex algebras associated to abelian current Lie algebras

In this talk, I will present the construction of a family of vertex algebras associated to the current algebra of finite-dimensional abelian Lie algebras. This family of vertex algebras and their module category are quasi-conformal and strongly $\mathbb{N}$-graded. I will verify certain important properties, such as convergence and extension property, needed in the logarithmic tensor category theory for strongly graded logarithmic modules developed by Huang, Lepowsky and Zhang.

Nina Yu, University of California – Riverside
Rationality, regularity and C_2-cofiniteness

Rationality, regularity and C_2-cofiniteness are three most important concepts in representation theory of vertex operator algebras. In this talk I will talk about connections among these three notions and recent progress in proving the conjecture that rationality implies C_2-cofinitenss.

Jian Zhang, North Carolina State University
Quantum determinants and quantum Pfaffians

We use quantum exterior algebras to give a new and elementary formulation of quantum Pfaffians and generalized quantum Pfaffians based on quantum Plucker relations. In this approach, the quantum Pfaffians are for any square matrix satisfying a simple quadratic relation. In particular, we prove the fundamental identity expressing any quantum determinant as a quantum Pfaffian.