Cosets of affine vertex algebras inside larger structures

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Joint work with T. Creutzig (University of Alberta),

Based on arXiv:1407.8512.

1. Orbifolds and cosets

Let $\mathcal{V}$ be a vertex algebra.

$G \subset \text{Aut}(\mathcal{V})$ a finite-dimensional, reductive group. Define *orbifold*

$$\mathcal{V}^G = \{v \in \mathcal{V} |\ gv = v, \quad \forall g \in G\}.$$

$\mathcal{A} \subset \mathcal{V}$ a vertex subalgebra. Define *coset*

$$\text{Com}(\mathcal{A}, \mathcal{V}) = \{v \in \mathcal{V} |\ [a(z), v(w)] = 0, \quad \forall a \in \mathcal{A}\}.$$

Suppose $\mathcal{V}$ has a nice property, such as strong finite generation, C_2-cofiniteness, or rationality.

Problem: Do $\mathcal{V}^G$ and $\text{Com}(\mathcal{A}, \mathcal{V})$ inherit this property?

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2. Classical invariant theory

G a finite-dimensional reductive group.

V a finite-dimensional G-module (over $\mathbb{C}$).

$\mathbb{C}[V]$ ring of polynomial functions on V.

$\mathbb{C}[V]^G$ ring of G-invariant polynomials.

Fundamental problem: Find generators and relations for $\mathbb{C}[V]^G$.

Thm: (Hilbert, 1893) $\mathbb{C}[V]^G$ is finitely generated for any G and V.

Basis theorem, Nullstellensatz, and syzygy theorem were all introduced by Hilbert in connection with this problem.

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3. First and second fundamental theorems

Let V be a G-module. For $j \geq 0$, let $V_j \cong V$. Let

$$R = \mathbb{C}[\oplus_{j \geq 0} V_j]^G.$$

First fundamental theorem (FFT) for (G, V) is a set of generators for R.

Second fundamental theorem (SFT) for (G, V) is a set of generators for the ideal of relations in R.

Some known examples:

- Standard representations of classical groups (Weyl, 1939)
- Adjoint representations of classical groups (Procesi, 1976),
- 7-dimensional respresentation of G_2 (Schwarz, 1988).

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4. Example: $G = \mathbb{Z}/2\mathbb{Z}$ and $V = \mathbb{C}$

Generator $\theta \in \mathbb{Z}/2\mathbb{Z}$ acts on V by -1.

x_j a basis for V_j^* for $j \geq 0$.

$\theta(x_j) = -x_j$.

$R = \mathbb{C}[\oplus_{j\geq 0} V_j]^{\mathbb{Z}/2\mathbb{Z}} = \mathbb{C}[x_0, x_1, x_2, \dots]^{\mathbb{Z}/2\mathbb{Z}}$ is the subalgebra of even degree.

FFT: R has quadratic generators $q_{i,j} = x_i x_j, \quad i \leq j$.

SFT: Relations are $q_{i,j} q_{k,l} - q_{i,k} q_{j,l}$.

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5. Example (Dong-Nagatomo, 1999)

Heisenberg vertex algebra $\mathcal{H}$ has generator $b(z)$ satisfying

$$b(z)b(w) \sim (z-w)^{-2}.$$

Basis $\{: \partial^{k_1} b \cdots \partial^{k_r} b : \mid 0 \leq k_1 \leq \cdots \leq k_r\}$.

$\mathrm{Aut}(\mathcal{H}) \cong \mathbb{Z}/2\mathbb{Z}$, generator $\theta : \mathcal{H} \to \mathcal{H}$ acts by $\theta(b) = -b$.

$\mathcal{H}$ is linearly isomorphic to $\mathbb{C}[x_0, x_1, x_2, \dots]$ where $x_j \leftrightarrow \partial^j b$.

Derivation $\partial(x_j) = x_{j+1}$.

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6. Example, cont'd

$R = \mathbb{C}[x_0, x_1, x_2, \dots]^{\mathbb{Z}/2\mathbb{Z}}$ has generators

$$q_{i,j} = x_i x_j, \qquad 0 \leq i \leq j.$$

Relations are $q_{i,j} q_{k,l} - q_{i,k} q_{j,l}$.

$R \cong \mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$, and $q_{i,j}$ correspond to strong generators for $\mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$:

$$\omega_{i,j} = :\partial^i b \partial^j b:, \qquad 0 \leq i \leq j.$$

Recall $\partial(q_{i,j}) = q_{i+1,j} + q_{i,j+1}$.

$\{q_{0,2k} | k \geq 0\}$ minimal generating set for R as a *differential algebra*.

$\{\omega_{0,2k} | k \geq 0\}$ strongly generates $\mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$. But this is *not* minimal!

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7. Example, cont'd

Thm: (Dong-Nagatomo, 1999) $\mathcal{H}^{\mathbb{Z}/2\mathbb{Z}}$ has minimal strong generating set $\{\omega_{0,0}, \omega_{0,2}\}$, and is of type $\mathcal{W}(2,4)$.

For all $k \geq 2$, we have a *decoupling relation* $\omega_{0,2k} = P(\omega_{0,0}, \omega_{0,2})$.

Ex :
$$\omega_{0,4} = -\frac{2}{5} :\omega_{0,0}\partial^2\omega_{0,0}: + \frac{4}{5} :\omega_{0,0}\omega_{0,2}: + \frac{1}{5} :\partial\omega_{0,0}\partial\omega_{0,0}: + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

Alternatively, this can be written in the form
$$\omega_{0,4} = -\frac{4}{5}(:\omega_{0,0}\omega_{1,1}: - :\omega_{0,1}\omega_{0,1}:) + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

This is a *quantum correction* of the analogous classical relation $q_{0,0}q_{1,1} - q_{0,1}q_{0,1}$.

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For all $k \geq 2$, we have a *decoupling relation* $\omega_{0,2k} = P(\omega_{0,0}, \omega_{0,2})$.

Ex : $$\omega_{0,4} = -\frac{2}{5} :\omega_{0,0}\partial^2\omega_{0,0}: + \frac{4}{5} :\omega_{0,0}\omega_{0,2}: + \frac{1}{5} :\partial\omega_{0,0}\partial\omega_{0,0}: + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

Alternatively, this can be written in the form

$$\omega_{0,4} = -\frac{4}{5}(:\omega_{0,0}\omega_{1,1}: - :\omega_{0,1}\omega_{0,1}:) + \frac{7}{5}\partial^2\omega_{0,2} - \frac{7}{30}\partial^4\omega_{0,0}.$$

This is a *quantum correction* of the analogous classical relation $q_{0,0}q_{1,1} - q_{0,1}q_{0,1}$.

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8. Free field algebras

Heisenberg algebra $\mathcal{H}(n)$: even generators $b^i, i = 1, \ldots, n$,

$$b^i(z)b^j(w) \sim \delta_{i,j}(z-w)^{-2}.$$

Free fermion algebra $\mathcal{F}(n)$: odd generators $\phi^i, i = 1, \ldots, n$,

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$\beta\gamma$-system $\mathcal{S}(n)$: even generators $\beta^i, \gamma^i, i = 1, \ldots, n$,

$$\beta^i(z)\gamma^j(w) \sim \delta_{i,j}(z-w)^{-1}.$$

Symplectic fermion algebra $\mathcal{A}(n)$: odd generators $e^i, f^i, i = 1, \ldots, n$,

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9. Orbifolds of free field algebras

$\mathcal{H}(n)$ and $\mathcal{F}(n)$ have full automorphism group $O(n)$.

$\mathcal{S}(n)$ and $\mathcal{A}(n)$ have full automorphism group $Sp(2n)$.

Thm: (L, 2012) $\mathcal{S}(n)^{Sp(2n)}$ is of type $\mathcal{W}(2,4,\dots,2n^2+4n)$.

Thm: (L, 2012) $\mathcal{F}(n)^{O(n)}$ is of type $\mathcal{W}(2,4,\dots,2n)$

Thm: (Creutzig-L, 2014) $\mathcal{A}(n)^{Sp(2n)}$ is of type $\mathcal{W}(2,4,\dots,2n)$.

Conj: (L, 2011) $\mathcal{H}(n)^{O(n)}$ is of type $\mathcal{W}(2,4,\dots,n^2+3n)$.

Thm: (L, 2012) This conjecture holds for $1 \leq n \leq 6$. For all n, $\mathcal{H}(n)^{O(n)}$ is strongly finitely generated (SFG).

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10. Orbifolds of free field algebras, cont'd

Thm: (L, 2012) Let $\mathcal{V}$ be either $\mathcal{H}(n)$, $\mathcal{F}(n)$, $\mathcal{S}(n)$, or $\mathcal{A}(n)$. For any reductive $G \subset \mathrm{Aut}(\mathcal{V})$, $\mathcal{V}^G$ is SFG.

Sketch of proof: For any reductive $G \subset \mathrm{Aut}(\mathcal{V})$, $\mathcal{V}^G$ is a module over $\mathcal{V}^{\mathrm{Aut}(\mathcal{V})}$.

By a theorem of Dong-Li-Mason (1996), $\mathcal{V}$ has a decomposition

$$\mathcal{V} = \bigoplus_{\nu \in S} L_\nu \otimes M_\nu.$$

L_ν ranges over all irreducible, finite-dimensional $\mathrm{Aut}(\mathcal{V})$-modules.

M_ν are inequivalent, irreducible $\mathcal{V}^{\mathrm{Aut}(\mathcal{V})}$-modules.

Zhu algebra of $\mathcal{V}^{\mathrm{Aut}(\mathcal{V})}$ is abelian, so each M_ν is highest-weight.

Using SFG property of $\mathcal{V}^{\mathrm{Aut}(\mathcal{V})}$, each M_ν is C_1-cofinite.

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11. Orbifolds of free field algebras, cont'd

$\mathcal{V}^G$ is also a direct sum of irreducible $\mathcal{V}^{\text{Aut}(\mathcal{V})}$-modules.

$\mathcal{V}^G$ has a generating set that lies in the direct sum of *finitely many* of these modules.

SFG property of $\mathcal{V}^G$ follows from these observations.

Let $\mathcal{V} = \mathcal{H}(n) \otimes \mathcal{F}(m) \otimes \mathcal{S}(r) \otimes \mathcal{A}(s)$ be a general free field algebra.

Let $G \subset \text{Aut}(\mathcal{V})$ be any reductive group preserving the tensor factors, i.e, $G \subset O(n) \times O(m) \times Sp(2r) \times Sp(2s)$.

Cor: $\mathcal{V}^G$ is SFG.

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12. Deformable families

$K \subset \mathbb{C}$ a subset which is at most countable.

F_K the $\mathbb{C}$-algebra of rational functions

$$\frac{p(\kappa)}{q(\kappa)}, \qquad \deg(p) \leq \deg(q),$$

such that the roots of q lie in K.

A *deformable family* $\mathcal{B}$ is a vertex algebra defined over F_K.

For $k \notin K$, ordinary vertex algebra $\mathcal{B}_k = \mathcal{B}/(\kappa - k)$.

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Thm: (Creutzig-L, 2012) A strong generating set for $\mathcal{B}_\infty$ gives rise to a strong generating set for $\mathcal{B}_k$ with the same cardinality, for generic values of k.

12. Deformable families

$K \subset \mathbb{C}$ a subset which is at most countable.

F_K the $\mathbb{C}$-algebra of rational functions

$$\frac{p(\kappa)}{q(\kappa)}, \qquad \deg(p) \leq \deg(q),$$

such that the roots of q lie in K.

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13. Examples

Let $\mathfrak{g}$ be a reductive Lie algebra with a nondegenerate form $\langle , \rangle$.

$V^k(\mathfrak{g})$ the corresponding universal affine vertex algebra.

Exists deformable family $\mathcal{V}$ with $K = \{0\}$ satisfying

$$\mathcal{V}_k = \mathcal{V}/(\kappa^2 - k) \cong V^k(\mathfrak{g}), \qquad k \neq 0.$$

$\mathcal{V}_\infty = \lim_{\kappa \to \infty} \mathcal{V} \cong \mathcal{H}(n)$, where $n = \dim(\mathfrak{g})$.

Let $G \subset \mathrm{Aut}(V^k(\mathfrak{g}))$ be a reductive group.

We have $\lim_{\kappa \to \infty} \mathcal{V}^G \cong \mathcal{H}(n)^G$.

Cor: $V^k(\mathfrak{g})^G$ is SFG for generic values of k.

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14. **Main result**

Let $\mathfrak{g}$ be a reductive Lie algebra of dimension n, with nondegenerate form $\langle , \rangle$.

Let $\mathfrak{g}' \supset \mathfrak{g}$ be a Lie algebra of dimension $m > n$, with nondegenerate form extending $\langle , \rangle$.

We have inclusion $V^k(\mathfrak{g}) \subset V^k(\mathfrak{g}')$.

Thm: (Creutzig-L, 2014) For all $\mathfrak{g}, \mathfrak{g}'$, $\mathcal{C}^k = \text{Com}(V^k(\mathfrak{g}), V^k(\mathfrak{g}'))$ is SFG for generic values of k.

Idea of proof: $\lim_{k\to\infty} \mathcal{C}^k \cong \mathcal{H}(m-n)^G$. Here G is a reductive group with Lie algebra $\mathfrak{g}$.

In some examples, can describe the set of nongeneric values of k. It is often finite or has compact closure.

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