Book of Abstracts

Another approach to Nevanlinna theory from diffusion processes

Atsushi Atsuji (Keio University, Japan)

Abstract:
We give another approach to Nevanlinna theory using diffusion processes. It gives generalizations of Nevanlinna theory for meromorphic functions on complete Kähler manifolds and some general manifolds. We apply this to value distribution of leafwise meromorphic functions on foliated manifolds with Kähler foliation. We consider Borel measurable maps from a compact foliated manifold with complex foliation to one dimensional complex projective space, which are holomorphic along the leaves. We call such maps leafwise meromorphic functions. We have a diffusion semigoup on the foliated manifold with respect to a harmonic measure introduced by L. Garnett. Assume the leaves are Kähler. Then we can use this semigroup to show some value distributional properties of such maps. In particular, ergodicity implies that nonconstant leafwise meromorphic functions can omit only a finite number of points under some conditions.

Surfaces with ample cotangent bundle

Damian Brotbek (Tokyo University)

Abstract:
Debarre conjectured that the cotangent bundle of a generic complete intersection variety of high codimension and high multidegree in projective space is ample. Via the theory of jet differentials, this conjecture is closely related to Kobayashi's conjecture on the hyperbolicity of generic hypersurfaces in projective space. By adapting a strategy towards Kobayashi's conjecture introduced by Siu and further developed by Diverio Merker and Rousseau, we will prove that Debarre's conjecture holds for surfaces."

Holomorphic extensions and the Monge-Ampere equation

Dan Burns (University of Michigan)

Abstract:
A famous result of Boutet de Monvel (1978) says that there exists a close connection between the maximal domain of holomorphic extension of a real analytic function into the complexification $X_{\mathbb{C}}$ of a closed real-analytic manifold X and the sub-level sets of solutions of homogeneous complex Monge-Ampère equations on the complexification. We discuss some examples of recent descendants of Boutet de Monvel's result, dealing with the maximal domain of such a complexification equipped with a Monge-Ampère solution, the algebraicization of certain Stein manifolds and the behavior of special metrics under the Kaehler-Ricci flow. These are joint results with R. Aguilar, V. Guillemin and Z. Zhang.

The $\overline{\partial}$-problem in product domains

Debraj Chakrabarti (Tata Institute, Bangalore, India)

Abstract:
We show that the $\overline{\partial}$-operator, in the L^2-sense, has closed range in a product domain provided it has closed range in each factor. We obtain regularity estimates for the canonical solution in special Sobolev spaces. We apply this technique to study the blowup of the canonical solution in the Hartogs Triangle. This is joint work with Mei-Chi Shaw.

Density of positive closed currents and dynamics of Hénon-type automorphisms of $\mathbb{C}^k$

Tien-Cuong Dinh (Institut de Mathématiques de Jussieu, France)

Abstract:
We introduce a new strategy to prove equidistribution properties in complex dynamics of several variables. We obtain the equidistribution for periodic points of Hénon-type maps on $\mathbb{C}^k$. A key point of the method is a notion of density which extends both the notion of Lelong number and the theory of intersection for positive closed currents on Kähler manifolds. This is a joint work with Nessim Sibony.

On Brody curves in P^n

Julien Duval (Universitë Paris - Sud, France)

Abstract:
We investigate the Brody curves of the projective space from the point of view of Nevanlinna theory.

Fubini–Study derivative of a holomorphic curve

Alexandre Eremenko (Purdue University)

Abstract:
For a holomorphic map f from the complex line to the projective space of dimension n, let $\|f'\|$ be the norm of the derivative with respect to the Euclidean metric on the complex line and the Fubini–Study metric on the projective space. If $\|f'(z)\| = O(|z|^\sigma)$, then the trivial (and best possible) estimate for the Nevanlinna characteristic is $T(r, f) = O(r^{2\sigma+2})$. However if f omits n hyperplanes in general position then this can be improved to $T(r, f) = O(r^{\sigma+1})$. This phenomenon was discovered by Clunie and Hayman in 1966 for the case $n = 1$. It is related to a conjecture of Littlewood (1952) about value distribution of entire functions. A proof of this result for arbitrary n will be discussed, as well as some other properties of the Fubini–Study derivatives of holomorphic curves. The talk is based on a joint work with Matthew Barrett.

On uniformly effective birationality and the Shafarevich Conjecture over curves

Gordon Heier (University of Houston)

Abstract:
We will discuss the following recent effective boundedness result for the Shafarevich Conjecture over function fields. Let B be a smooth projective curve of genus g, and $S \subset B$ be a finite subset of cardinality s. There exists an effective upper bound on the number of deformation types of admissible families of canonically polarized manifolds of dimension n with canonical volume v over B with prescribed degeneracy locus S. The effective bound only depends on the invariants g, s, n and v. The key new

ingredient which allows for this kind of result is a careful study of effective birationality for families of canonically polarized manifolds. This is joint work with S. Takayama.

Hyperbolicity in a group action setting

Alan Huckleberry (Ruhr-Universität Bochum)

Abstract:
Our view of a certain standard representation theoretic setup, where one considers actions of a real Lie group on flag manifolds of its complexification, has been strongly influenced by the fact that certain associated cycle spaces are hyperbolic. This connection and several recent applications will be explained.

Our discussion will be intermingled with recollections of Wilhelm Stoll and Pit-Mann Wong.

Weak Geodesics in the Space of Kähler Metrics

László Lempert (Purdue University)

Abstract:

Given a compact Kähler manifold (X, ω_0), according to Mabuchi, the set $\mathcal{H}_0$ of Kähler forms cohomologous to ω_0 has the natural structure of an infinite dimensional Riemannian manifold. We address the question whether points in $\mathcal{H}_0$ can be joined by a geodesic, and strengthening our earlier findings with Liz Vivas, we show that this cannot always be done even with a certain type of generalized geodesics. As in the work with Vivas, the result is obtained through the analysis of a Monge–Ampère equation.

Weak Geodesics in the Space of Kähler Metrics

Joël Merker (Orsay University, France)

Abstract:

For every smooth algebraic curve $X^1 \subset \mathbb{P}^2(\mathbb{C})$ represented as $\{0 = R(x,y)\}$ in some affine $\mathbb{C}^2_{x,y} \subset \mathbb{P}^2$, the $(x \leftrightarrow y)$-symmetric equation:

$$
\begin{aligned}
&\frac{x'''}{R_y} + \frac{3x'x''}{R_y}\left[\left(\frac{R_x}{R_y}\right)\frac{R_{yy}}{R_y} - \frac{R_{xy}}{R_y}\right] + \frac{(x')^3}{R_y}\left[3\left(\frac{R_x}{R_y}\right)^2\frac{R_{yy}}{R_y}\frac{R_{yy}}{R_y} - 6\left(\frac{R_x}{R_y}\right)\frac{R_{xy}}{R_y}\frac{R_{yy}}{R_y} + \right.\\
&\qquad \left. + 3\frac{R_{xy}}{R_y}\frac{R_{xy}}{R_y} + 3\frac{R_{xx}}{R_y}\frac{R_{yy}}{R_y} - \left(\frac{R_x}{R_y}\right)^2\frac{R_{yyy}}{R_y} + 3\left(\frac{R_x}{R_y}\right)\frac{R_{xyy}}{R_y}\right] = \\
&= -\frac{y'''}{R_x} - \frac{3y'y''}{R_x}\left[\left(\frac{R_y}{R_x}\right)\frac{R_{xx}}{R_x} - \frac{R_{xy}}{R_x}\right] - \frac{(y')^3}{R_x}\left[3\left(\frac{R_y}{R_x}\right)^2\frac{R_{xx}}{R_x}\frac{R_{xx}}{R_x} - 6\left(\frac{R_y}{R_x}\right)\frac{R_{xy}}{R_x}\frac{R_{xx}}{R_x} + \right.\\
&\qquad \left. + 3\frac{R_{xy}}{R_x}\frac{R_{xy}}{R_x} + 3\frac{R_{yy}}{R_x}\frac{R_{xx}}{R_x} - \left(\frac{R_y}{R_x}\right)^2\frac{R_{xxx}}{R_x} + 3\left(\frac{R_y}{R_x}\right)\frac{R_{xxy}}{R_x}\right]
\end{aligned}
$$

and its higher jet generalizations describe in a purely effective way holomorphic jet differentials on X^1, always with only $\frac{1}{R_y}$ on the left, and only $\frac{1}{R_x}$ on the right.

For $X^2 \subset \mathbb{P}^3$ smooth, starting from:

$$
\begin{aligned}
\frac{y'}{R_z} &= -\frac{x'}{R_y}\frac{R_x}{\underline{R_z}} - \frac{z'}{R_y},\\
\frac{x'y'' - y'x''}{R_z} &= \frac{z'x'' - x'z''}{R_y} - \frac{(x')^3}{R_y}\left[\frac{R_{xx}}{\underline{R_z}} - 2\frac{R_x}{R_y}\frac{R_{xy}}{\underline{R_z}} + \left(\frac{R_x}{R_y}\right)^2\frac{R_{yy}}{\underline{R_z}}\right] - \\
&\qquad - \frac{2(x')^2z'}{R_y}\left[\frac{R_{xz}}{\underline{R_z}} - \frac{R_{xy}}{R_y} - \frac{R_x}{R_y}\frac{R_{yz}}{\underline{R_z}} + \frac{R_x}{R_y}\frac{R_{yy}}{R_y}\right] - \\
&\qquad - \frac{x'(z')^2}{R_y}\left[\frac{R_{zz}}{\underline{R_z}} - 2\frac{R_{yz}}{R_y} + \frac{R_z}{R_y}\frac{R_{yy}}{R_y}\right],
\end{aligned}
$$

one should find analogous symmetric jet polynomials, though of weighted degree $\geqslant 10$. A new, systematic, *non-linear*, constructive Čech-cohomology theory would lead to differential-algebraic cochains which would supersede coarse asymptotic cohomology inequalities.

At least, for $X^n \subset \mathbb{P}^{n+1}$, explicit jet differentials that are holomorphic on an affine chart, but have poles on $X \cap \mathbb{P}^n_\infty$, may be shown to exist. Also, explicit sections of Schur bundles $\mathcal{S}^{(\ell_1,\dots,\ell_n)}T^*_X$ that are holomorphic on the whole of X may be shown to exist whenever $\ell_n \geqslant 1$.

As soon as (partially) explicit jet differentials are constructed (not yet so!), Kobayashi hyperbolicity of generic high degree smooth hypersurfaces $X^n \subset \mathbb{P}^{n+1}(\mathbb{C})$ follows by

adapting Siu's slanted vector fields argument, but *without inducting* on dimension, because base loci of entire curves are then explicitly controlled in the quotient ring of the algebra in jets of the defining polynomial.

Entire curves into semi-abelian varieties and related topics

Junjiro Noguchi (University of Tokyo, Japan)

Abstract:
We will recall the S.M.T. of entire curves into semi-abelian varieties with truncation level one, and discuss its applications. For example, we deduce the algebraic degeneracy of entire curves in a projective algebraic variety V of general type with irregularity $q(V) \geq \dim V$ (in fact, $\kappa(V) > 0$ suffices in this case); this is, so far, the best general result for Green-Griffiths' conjecture only in terms of numerical invariants. We also dicuss an application to Yamanoi's unicity theorem, and also some arithmetic analogue. If time allows, we will talk about some new phenomena in value distribution theory such that the image space being kähler or non-kähler makes a difference, and discuss some questions.

Green Pluripotential On Almost Complex Domains

Giorgio Patrizio (Università degli Studi di Firenze, Italy)

Abstract:
Starting with ideas on Monge-Ampère foliations rooted in the work of Wilhelm Stoll and Pit-Mann Wong, I will outline recent results obtained with Andrea Spiro about the construction of generalized Riemann maps and normal forms for almost complex domains with singular foliations by stationary disks. Such normal forms are used to construct examples and to determine intrinsic conditions under which stationary disks are extremal for the Kobayashi metric and allow the construction of Green pluripotential in the almost complex setting.

On the hyperbolicity of surfaces of general type with small K^2.

Erwan Rousseau (CMI, Université de Provence)

Abstract:
We study the hyperbolicity of surfaces of general type with minimal K^2, known as Horikawa surfaces. (Joint work with X. Roulleau).

Nevanlinna theory and its applications

Min Ru (University of Houston)

Abstract:
To dedicate my talk to the memory of Professors Wilhelm Stoll and Pit-Mann Wong, I'll talk about some of my research work along the three directions which were initiated during my graduate study at Notre Dame, supervised by Professors Stoll and Wong: 1. Cartan's Second Main Theorem and beyond (the Second Main Theorem with moving targets, the Second Main Theorem with hypersurfaces, ...); 2. Nevanlinna theory and Diophantine approximation (Integer solutions of Polynomial equations and inequalities); 3. Value distribution of Gauss maps of minimal surfaces in R^n.

Einstein metrics on Kähler manifolds

Yanir Rubinstein (Stanford University)

Abstract:
The Uniformization Theorem implies that any compact Riemann surface has a constant curvature metric. Kähler-Einstein (KE) metrics are a natural generalization of such metrics, and the search for them has a long and rich history, going back to Schouten, Kähler (30's), Calabi (50's), Aubin, Yau (70's) and Tian (90's), among others. Yet, despite much progress, a complete picture is available only in complex dimension 2.

In contrast to such smooth KE metrics, in the mid 90's Tian conjectured the existence of KE metrics with conical singularities along a divisor (i.e., for which the

manifold is 'bent' at some angle along a complex hypersurface), motivated by applications to algebraic geometry and Calabi-Yau manifolds. More recently, Donaldson suggested a program for constructing smooth KE metrics of positive curvature out of such singular ones, and put forward several influential conjectures.

In this talk we will try to give an introduction to Kähler-Einstein geometry and briefly describe some recent work mostly joint with R. Mazzeo that resolves some of these conjectures. One key ingredient is a new $C^{2,\alpha}$ a priori estimate and continuity method for the complex Monge-Ampere equation. It follows that many algebraic varieties that may not admit smooth KE metrics (e.g., Fano or minimal varieties) nevertheless admit KE metrics bent along a simple normal crossing divisor.

Moduli of Riemann Surfaces: A View from Kähler Geometry

Georg Schumacher (Philipps-Universität Marburg, Germany)

Abstract:
We apply methods of Kähler geometry to moduli spaces of Riemann surfaces and give an outlook to moduli of varieties of higher dimension. We use methods of the theory of Kähler -Einstein manifolds and study the first and second variation of the geodesic length function and twist parameters for families of compact Riemann surfaces. Applications concern plurisubharmonic exhaustion functions, the equivalence of the Fenchel-Nielsen symplectic and Weil-Petersson forms and positive line bundles on moduli spaces.

L^p norms of random holomorphic sections

Bernard Shiffman (Johns Hopkins University)

Abstract:
This talk concerns the distribution of values of L^p norms of holomorphic sections s_N of powers L^N of a positive line bundle L over a compact Kähler manifold M, for $2 < p \leq \infty$. If s_N has unit L^2 norm, then its L^p norm can be quite large; indeed, its maximum value is $\sim N^{m(1/2-1/p)}$. However, such large values are rare events; almost all sequences of L^2-normalized sections s_N have uniformly bounded L^p norms for $p < \infty$, and have L^∞ norms growing like $\sqrt{\log N}$. Furthermore, we show how to construct these

sequences so that the L^∞ norms are also uniformly bounded. The methods involve the off-diagonal scaling asymptotics of the Szegő kernel for $H^0(M, L^N)$.

Second Main Theorem and Holomorphic Dynamics.

Nessim Sibony (Orsay University)

Abstract:
I will discuss some analogies between the second main theorem in Nevanlinna's theory and results in holomorphic dynamics i.e foliations by Riemann Surfaces and discrete holomorphic dynamics.

This is joint work with T.C Dinh.

Partial Frobenius integrability and its applications

Yum-Tong Siu (Harvard University)

Abstract:
Will start with the work of Frobenius in 1877 on integrability of distribution of tangent subspaces and trace the development of partial Frobenius integrability through the work of Caratheodory on the second law of thermodynamics in 1909, Chow Wei-liang's generalization in 1939, and Hormander's work on sums of squares of vector fields in 1967. Then will discuss recent applications to Kohn's conjecture on subellipticity for finite-type domains and to the abundance conjecture in algebraic geometry.

On the Demailly-Semple jet bundles of hypersurfaces in the 3-dimensional complex projective space

JingZhou Sun (Johns Hopkins University)

Abstract:
Let X be a smooth hypersurface of degree d in the 3-dimensional complex projective space. By totally algebraic calculations, we prove that on the third Demailly-Semple jet bundle X_3 of X, the Demailly-Semple line bundle is big for $d \geq 11$, and that on the fourth Demailly-Semple jet bundle X_4 of X, the Demailly-Semple line bundle is big for $d \geq 10$, improving a recent result of Diverio.

Curvature of fields of quantum Hilbert spaces

Róbert Szőke (Eötvös Loránd University, Hungary)

Abstract:
This is joint work with Laszlo Lempert. For a Riemannian manifold M (in good cases) there exists a canonical complex structure on its tangent bundle. This is characterised by the property that the leaves of the Riemann foliation in TM are complex submanifolds. Recently we discovered that it is advantageous to take the phase space N to be rather the set of parameterized geodesics in M. Then this complex structure in fact becomes just one element of a canonical natural family of complex structures on N, parameterized by the upper half plane S. The procedure of geometric quantization assigns a (so called) quantum Hilbert space to a Kähler manifold with integral Kähler form, namely the L2-holomorphic sections of an appropriate holomorphic line bundle. In our situation geometric quantization performed on this family of Kähler structures produces a set of Hilbert spaces parameterized by S. The main question here is how does this family of quantum Hilbert spaces depends on the parameters. To be able to answer such questions (due to the technical difficulty of N to be noncompact) we introduced a generalization of Hilbert bundles, that we call field of Hilbert spaces. It is possible to introduce smooth and analytic structure on such fields in question and talk about their curvature. It can be proved that flat fields over S can be trivialized ie our field in this case is an honest Hilbert bundle, its parallel translation canonically identifies the different fibers with unitary maps, showing that geometric quantization is unique in such cases. Slightly more generally when the curvature is projectively flat

(ie the curvature is pointwise a constant multiple of identity), after a simple twisting reduces to the flat case and we obtain a canonical (up to a scalar factor) unitary identifications of the fibers of the field in question. In this talk we plan to explain the following theorem. When M is a simply connected, compact rank-1 symmetric space, the field of quantum Hilbert spaces, using the natural family of adapted Kähler structures, is flat when M= the 3-sphere and not even projectively flat in the other cases.

Kobayashi Hyperbolic Imbedding into Toric Varieties

Yusaku Tiba (University of Tokyo, Japan)

Abstract:
Our main goal of this talk is to give a characterization of an algebraic divisor on an algebraic torus whose complement is Kobayashi hyperbolically imbedded into a toric projective variety. As an application of our main theorem, we prove the following: the complement of the union of $n+1$ hyperplanes in the n-dimensional projective space $\mathbb{P}^n(\mathbb{C})$ in general position and a general hypersurface of degree n in $\mathbb{P}^n(\mathbb{C})$ is Kobayashi hyperbolically imbedded into $\mathbb{P}^n(\mathbb{C})$.

Basins of attraction of automorphisms in $\mathbb{C}^3$

Liz Vivas (Purdue University)

Abstract:
I will talk about a construction of an automorphism of $\mathbb{C}^3$ with a region of attraction that is not simply connected. This is joint work with Berit Stensønes.

The McQuillan-Chen proof of the “1+epsilon” conjecture over function fields

Paul Vojta (University of California, Berkeley)

Abstract:
The “1+epsilon” conjecture is the counterpart in number theory to the Second Main

Theorem for maps from proper surjective covers of the complex plane to compact Riemann surfaces. If proved, it would imply the abc conjecture and several other outstanding conjectures in number theory. In a 2005 IHES preprint, M. McQuillan proved this conjecture for function fields of characteristic zero. His proof relies heavily on functional analysis, and is not easy to understand. Subsequently, Xi Chen (Can. J. Math. 2011) gave a proof that eliminates the functional analysis, instead using methods from algebraic geometry over C. This paper has some mistakes, though. All of these have been fixed by Chen. I will describe this (revised) proof.

Second Main Theorems and Büchi's Problem over Function Fields

Julie Tzu-Yueh Wang (Academia Sinica - Taiwan)

Abstract:
Hilbert's Tenth Problem asks whether there is a general algorithm to determine, given any polynomial in several variables, whether there exists a zero with all coordinates in $\mathbf{Z}$. It was proved in the negative by Yu. Matiyasevich in 1970. In the 70's J. R. Büchi attempted to prove a similar statement for a system of quadric equations, and he was able to relate it to the following Diophantine problem:

Conjecture (Büchi's square problem). *There exists an integer $M > 0$ such that all $x_1, ..., x_M \in \mathbf{Z}$ satisfying the equations*

$$x_1^2 - 2x_2^2 + x_3^2 = x_2^2 - 2x_3^2 + x_4^2 = \cdots = x_{M-2}^2 - 2x_{M-1}^2 + x_M^2 = 2$$

must also satisfy $x_i^2 = (x+i)^2$ for a fixed integer x and $i \in \{1, ..., M\}$. A generalization of Büchi's square problem asks is there a positive integer integer M such that any sequence $(x_1^n, ..., x_M^n)$ of n-th powers of integers with n-th difference equal to $n!$ is necessarily a sequence of n-th powers of successive integers.

In this talk, we will first introduce some second main theorems over function fields and then use them to study generalized Büchi's problem for algebraic functions. The method also works for meromorphic functions and non-archimedean meromorphic functions.

Complete Kähler-Einstein metrics on quasi-projective manifolds revisited

Damin Wu (Ohio State University)

Abstract:
It is known that a quasi-projective manifold with ample logarithmic canonical bundle admits a canonical, complete Kähler-Einstein metric. The asymptotic behavior of the canonic metric has been studied in the literature. In this talk I will present a more precise asymptotic expansion of the canonic metric near the boundary divisor. This result would imply a necessary condition on the existence of complete Kähler-Einstein metrics on more general quasi-projective manifolds.

Realizing Serre duality as a product of currents

Elizabeth Wulcan (Chalmers University of Technology, Sweden)

Abstract:
I will discuss a joint work in progress with Hkan Samuelsson Kalm and Jean Ruppenthal. Given an analytic space X, we define (by modifying a recent construction by Andersson-Samuelsson Kalm) fine sheaves $\mathcal{A}_X^{0,q}$ of $(0,q)$-currents that are smooth on the regular part of X and that give a resolution of the structure sheaf $\mathcal{O}_X$. If X is Gorenstein we also introduce sheaves $\mathcal{A}_X^{n,q}$ of (n,q)-currents that give a resolution of the dualizing sheaf ω_X. The sheaf $\mathcal{A}_X^{*,*}$ has a multiplicative structure; there is a well-defined wedge product $\mathcal{A}_X^{p,q} \times \mathcal{A}_X^{p',q'} \to \mathcal{A}_X^{p+p',q+q'}$, which gives a pairing $H^{n-q}(X,\mathcal{O}_X) \times H^q(X,\omega_X) \to \mathbb{C}$ by $([\alpha],[\beta]) \mapsto \int \alpha \wedge \beta$. This pairing is non-degenerate and thus realizes Serre duality on X.

Global property for local holomorphic isometries between compact Hermitian symmetric spaces

Yuan Yuan (Johns Hopkins University)

Abstract:
Motivated by recent work of Mok and others on local holomorphic isometric embeddings, we consider local conformal maps from an irreducible Hermitian symmetric space of compact type into the products of such manifolds. We allow the conformal factors to be arbitrary real numbers and derive a necessary and sufficient condition for the global rigidity to hold. This is a joint work with X. Huang.

Global extension and Rigidity for local holomorphic isometric embeddings

Yuan Zhang (University of California - San Diego)

Abstract:
In this talk, we discuss local holomorphic isometric embeddings from $\mathbb{B}^n$ into $\mathbb{B}^{N_1}$ x $\cdots$ x $\mathbb{B}^{N_m}$ with respect to the normalized Bergman metrics up to conformal factors. Assume that each conformal factor is smooth Nash algebraic and $n \geq 2$. Then each component of the map is a multi-valued holomorphic map between complex Euclidean spaces by the algebraic extension theorem derived along the lines of Mok and Mok-Ng. Applying holomorphic continuation together with a linearity criterion of Huang, we conclude the total geodesy of non-constant components. This is a joint work with Yuan Yuan.