

**Complex Geometry and Several Complex  
Variables at The University of Notre Dame  
March 9-11, 2018  
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**BOOK OF ABSTRACTS**

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**Dan Burns (University of Michigan)**

*Kobayashi disks and twistors*

Abstract: Penrose's twistor theory associates metrics and conformal classes of metrics to manifolds of four real dimensions which are moduli spaces of rational curves in a complex three manifold. LeBrun and Mason have been exploring the case of the moduli spaces of certain families of holomorphic disks, extending Penrose's work. Some time ago Lempert gave a thorough description of the Kobayashi disks of a smoothly bounded, strictly convex domain  $D \subset \mathbb{C}^n$ , as well as a dual problem on the exterior of the dual set  $\hat{D} \subset \hat{\mathbb{P}}^2$ . We discuss work, joint with John Bland and Kin-Kwan Leung, extending the work of these authors where the family of disks to be studied are the Kobayashi disks of the strictly convex domain  $D$ . This gives a four dimensional moduli space  $X$  with an anti-self dual metric of signature (2,2). (The LeBrun-Mason construction leads to self-dual four manifolds of signature (2,2).) The identification of  $X$  appears somewhat subtle, but the construction suggests strong parallels with classical projective duality. Several questions remain open, dealing, e.g., with extended families of holomorphic disks in our context.

**Tristan Collins (Harvard University)**

*The Inverse Monge-Ampère flow*

Abstract: I will discuss a new parabolic Monge-Ampère equation with applications to the existence (and non-existence) of Kähler-Einstein metrics. I will discuss the long-time existence of the flow, and convergence on Kähler-Einstein manifolds. Time permitting, I will also discuss the convergence of the flow to the maximally destabilizing degeneration on toric Fano manifolds. This is joint work with T. Hisamoto, and R. Takahashi.

**Dan Coman (Syracuse University)**

*Equidistribution and universality results for sequences of line bundles on compact Kaehler spaces*

Abstract: We study the asymptotic distribution of the Fubini-Study currents associated to a sequence of singular Hermitian holomorphic line bundles on a compact normal Kaehler complex space. This is a generalization of our earlier results, by allowing the base space to be singular, and by considering sequences of line bundles instead of the sequence of powers of a fixed line bundle.

We also discuss universality results in the above setting, which show that, under mild moment assumptions, the asymptotic distribution of zeros of random sequences of holomorphic sections is independent of the choice of probability measures on the spaces of holomorphic sections.

The results are joint work with Xiaonan Ma and George Marinescu, and with Turgay Bayraktar and George Marinescu.

**Eleonora Di Nezza (Institut des Hautes Études Scientifiques, France)**

*On the dimension of the Bergman space on some unbounded domains*

Abstract: In this talk we present a proof of the log-concavity property of total masses of positive currents on a given compact Kähler manifold, that was conjectured by Boucksom, Eyssidieux, Guedj and Zeriahi. The proof relies on the resolution of complex Monge-Ampère equations with prescribed singularities. This is based on a joint work with Tamas Darvas and Chinh Lu.

**Peter Ebenfelt (University of California - San Diego)**

*The obstruction function on strictly pseudoconvex hypersurfaces in  $\mathbb{C}^2$*

Abstract: The obstruction function is a CR invariant that arises as the obstruction for the Cheng-Yau solution (whose log is the potential of the Poincaré-Einstein metric in a strictly pseudoconvex smoothly bounded domain  $\Omega$ ) to extend smoothly to the boundary  $\partial\Omega$ . In  $\mathbb{C}^2$ , it also coincides (modulo a non-zero constant factor) with the restriction to the boundary of the log term in the Bergman kernel. This invariant vanishes if the boundary is locally spherical, and it is an open question whether the converse holds. We shall discuss some recent results regarding this question, and related ones for deformations.

**Siqi Fu (Rutgers University-Camden)**

*Asymptotic spectral behavior of the  $\bar{\partial}$ -Neumann Laplacian*

Abstract: Asymptotic distribution of the eigenvalues for the  $\bar{\partial}$ -Neumann Laplacian on strictly pseudoconvex domains in  $\mathbb{C}^n$  has been studied extensively by Nancy Stanton and others in the 1980's and is well understood because of their work. It is also known that for a smooth bounded pseudoconvex domain in  $\mathbb{C}^2$ , spectral asymptotics can be used to characterize the finite type condition. In this talk, we review these results and discuss relevant recent developments.

**Anne-Katrin Gallagher (Oklahoma State University)**

*Log-concavity of the volume of positive currents*

Abstract: A sufficient condition for the infinite dimensionality of the Bergman space of a pseudoconvex domain is given. This condition holds on any pseudoconvex domain that has at least one smooth boundary point of finite type in the sense of D'Angelo. This is joint work with T. Harz and G. Herbort.

**Purvi Gupta (Rutgers University)**

*Polynomially convex embeddings of compact real manifolds*

Abstract: In this talk, we will discuss some questions regarding the minimum embedding (complex) dimension of abstract compact (real) manifolds within the context of polynomial convexity and polynomial approximations. In the special case of even-dimensional manifolds, we will present a technique that improves previously known bounds. This is joint work with Rasul Shafikov.

**Phillip Harrington (University of Arkansas)**

*The Diederich-Fornaess Index and Global Regularity*

Abstract: We say that the Bergman Projection is globally regular on a given domain if it preserves the space of functions that are smooth up to the boundary on this domain. In this talk, we will examine sufficient conditions for global regularity of the Bergman Projection. We will focus on recent work involving the Diederich-Fornaess Index. The Diederich-Fornaess Index for a given domain is the supremum over all exponents  $0 < s < 1$  such that there exists a defining function  $r$  for the domain with the property that  $-(-r)^s$  is plurisubharmonic on the domain. There appears to be a close connection between domains on which the Bergman Projection is globally regular and domains on which the Diederich-Fornaess Index is equal to one, but the precise nature of this connection remains elusive.

**Christine Laurent-Thiébaud (Fourier Institute)**

*About the Dolbeault cohomology with prescribed support*

Abstract: After noting that the Dolbeault isomorphism holds no more between the Dolbeault cohomology groups with prescribed support in the closure of a domain of a complex manifold and values in different spaces, we will give some characterizations of pseudoconvexity using these groups. More precisely for a bounded domain  $D$  of the Euclidean space, depending on the regularity of the boundary of  $D$ , we will characterize the pseudoconvexity of  $D$  using the vanishing and Hausdorff properties of the Dolbeault cohomology groups with prescribed support in the closure of  $D$  and values in the right space (This is based on joint works with Siqi Fu and Mei-Chi Shaw).

**Lázló Lempert (Purdue University)**

*On the geometry of the space of Kahler metrics*

Abstract: On a fixed Kähler manifold all possible Kähler metrics form an infinite dimensional manifold  $H$ . In the 1980s Mabuchi introduced a Riemannian metric on  $H$ , and computed its curvature. The computation suggested that  $H$  with its Mabuchi metric is a so called locally symmetric space. I will discuss what this suggestion means, and to what extent it is justified.

**Min Ru (University of Houston)**

*Holomorphic curves into projective varieties intersecting general divisors*

Abstract: We establish a general Second Main Theorem for holomorphic curves into the projective variety  $X$  intersecting general divisor  $D$ , in terms of the (birational) Nevanlinna constant  $Nev_{bir}(D)$ . By computing  $Nev_{bir}(D)$  using the filtrations, it recovers (almost all) previous known results in this direction, as well as derive some new results for divisors which are not necessarily linear equivalent on  $X$ . The notion  $Nev_{bir}(D)$  is originally defined in terms of Weil functions for use in applications, and it is proved later that it can be defined in terms of local effectivity of Cartier divisors after taking a proper birational lifting. This is a joint work with Paul Vojta.

**Liz Vivas (Ohio State University)**

*Towards a topological classification of parabolic skew-product maps*

Abstract: We consider skew product maps of the form  $F(z, w) = (h(z), f(z, w))$ . We deal with the case of parabolic skew product maps, that is when  $DF(0, 0) = Id$ . Our goal is to describe the behavior of orbits around a whole neighborhood of the origin. We will present the relevant results on one dimension and explain the obstacles on higher dimensions.

**Ming Xiao (University of California - San Diego)**

*Rigidity of volume-preserving maps between Hermitian symmetric spaces*

Abstract: We discuss rigidity results of volume preserving maps between Hermitian symmetric spaces, based on the work of Mok-Ng and my recent joint work with Fang and Huang. Moreover, we make connections with rigidity results in CR geometry.

**Paul Yang (Princeton University)**

*CR invariant energy for surfaces and curves in the standard 3-sphere*

Abstract: I plan to present two invariant energy functionals for surfaces in the 3-sphere. One of which appears as the obstruction for smoothness solutions to a CR analogue of the Lowner-Nirenberg problem.

**Andrew Zimmer (College of William and Mary)**

*The automorphism group and limit set of a bounded domain*

Abstract: For certain classes of bounded domains we give a precise description of the automorphism group and its limit set in the boundary. For instance, for finite type pseudoconvex domains we prove: if the limit set contains at least two different points, then the automorphism group has finitely many components and is the almost direct product of a compact group and a connected Lie group locally isomorphic to the automorphism group of a ball. Further, the limit set is a smooth submanifold diffeomorphic to a sphere. For convex domains with  $C^{1,\epsilon}$  boundary we can prove a similar result. In this talk we will describe some ideas in the proof and discuss some applications.