

Cluster Structures in Geometry, Physics, Combinatorics and Representation Theory

*University of Notre Dame's Jerusalem Global Gateway
Tantur Ecumenical Institute*

December 9 – 16, 2018

SCHEDULE AND ABSTRACTS

Monday December 10, 2018

9:30 AM **Vladimir Fock (Université de Strasbourg)**
Cluster Structure of Discrete Integrable Systems

11:00 AM **Philippe Di Francesco (University of Illinois at Urbana-Champaign)**
Macdonald Operators and Cluster Algebra

14:30 PM **Anna Felikson (Durham University)**
Mutations of Non-Integer Quivers: Finite Mutation Type

Given a skew-symmetric non-integer (real) matrix, one can construct a quiver with non-integer weights of arrows. Such a quiver can be mutated according to usual rules of quiver mutation. We classify non-integer quivers of finite mutation type and prove that all of them admit some geometric interpretation (either related to orbifolds or to reflection groups). In particular, the reflection group construction gives rise to the notion of quivers of finite and affine types. We also study exchange graphs of quivers of finite and affine types in rank 3. The talk is based on joint works with Pavel Tumarkin and Philipp Lampe.

16:00 PM **Kathryn Mulholland and Danny Orton (University of Notre Dame)**
Generalized Cluster Structures Compatible with the Cremmer-Gervais Poisson Bracket on Rectangular Matrices

Gekhtman, Shapiro, and Vainshtein have conjectured that there exists a classification of regular cluster structures on simple complex Lie groups that is parallel to the Belavin-Drinfeld classification. In our talk, we will outline a construction of an initial cluster for a generalized cluster structure on the space of rectangular $m \times n$ matrices (that is compatible with the restriction of the Cremmer-Gervais Poisson bracket on GL_n) and investigate conditions under which there exists a sequence of cluster mutations relating cluster structures on $Mat_{m \times n}$ and $Mat_{(m-1) \times n}$.

Tuesday December 11, 2018

9:30 AM **Emine Yildirim (Queen's University)**
Associahedra and Newton Polytopes of F-Polynomial

Generalized associahedron is a polytope whose outer normal fan is a g -vector fan. Such polytopes get considerable attention since their combinatorics encodes the structure of a cluster algebra. Recently, Arkani-Hamed, Bai, He, and Yan (2017) gave a new approach for constructing associahedra in connection with the physics of scattering forms. Their construction can be viewed as constructing an associahedron from a linearly oriented type A Dynkin diagram. We generalize their procedure of constructing an associahedron to all simply-laced Dynkin diagrams using quiver

representations. Furthermore, we show that our construction realizes the Newton polytopes of the F-polynomials of the corresponding cluster algebras. This is a joint work with Véronique Bazier-Matte, Guillaume Douville, Kaveh Mousavand, and Hugh Thomas.

11:00 AM **Vincent Pilaud (CNRS École Polytechnique)**

On Type Cones of G-Vector Fans

In a recent preprint, Bazier-Matte -- Douville -- Mousavand -- Thomas -- Yildirim constructed all polytopal realizations of the g-vector fans of finite type cluster algebras with respect to an acyclic initial seed. This talk will explain the geometric reason hidden behind their construction and will propose an alternative proof of their result. More generally, we will discuss the type cone (in other words, the space of all polytopal realizations) of g-vector fans of finite type cluster-like complexes (finite type cluster complexes, non-kissing complexes of gentle algebra, graph associahedra). Joint work in progress with Arnau Padrol, Yann Palu and Pierre-Guy Plamondon.

14:30 PM **Nathan Reading (North Carolina State University)**

A Combinatorial Approach to Cluster Scattering Diagrams

In finite type, the combinatorics of a cluster algebra is encoded in its g-vector fan. In infinite type, the g-vector fan still describes the combinatorics of clusters, but fails to fully capture the combinatorics of additive bases for the cluster algebra. Recently, Gross, Hacking, Keel, and Kontsevich constructed cluster scattering diagrams and used them to prove many of Fomin and Zelevinsky's structural conjectures on cluster algebras. They showed that the cluster scattering diagram (a collection of codimension-1 cones) "cuts out" the g-vector fan. More recently, I showed that the cluster scattering diagram cuts out a complete fan containing the g-vector fan as a subfan. This fan is the most general "generalized associahedron fan", the combinatorial essence of a general cluster algebra. In particular, it appears to be the right combinatorial scaffolding upon which to build additive bases.

The cluster scattering diagram arises from a non-constructive (in practice) existence theorem. Thus combinatorial models for cluster scattering diagrams and fans are needed. After a brief digression about the combinatorics of cluster scattering diagrams of affine rank 2, I will describe work in progress to construct cluster scattering diagrams and fans in affine type (with Stella) and in the surfaces case (with Muller and Viel).

16:00 PM **Chris Fraser (University of Minnesota)**

Tensor Diagrams and Cluster Variables on Punctured Surfaces

We propose a conjectural combinatorial description of cluster variables and clusters for spaces SL_k -local systems on a bordered surface in the presence of punctures, generalizing the notion of "tagged arc complex" for cluster algebras from surfaces. As evidence for our conjecture, we will prove it in some nontrivial cases. Joint with Pavlo Pylyavskyy.

Wednesday December 12, 2018

FREE DAY

Thursday December 13, 2018

9:30 AM **Ilke Canakci (Newcastle University)**

Generalised Friezes and the Weak Ptolemy Map

The notion of frieze pattern was introduced by Conway and subsequently Conway--Coxeter showed that triangulations of polygons are in bijection with frieze patterns of positive integers.

Due to their connection to cluster algebras and cluster categories, there is a renewed interest on frieze patterns and several generalisations have been established. One generalisation of interest is that of associated to polygon dissections; i.e. to partial triangulations of polygons. In joint work with Peter Jørgensen, we introduce a 'weak Ptolemy map' from diagonals of polygons to any ring and show that this gives rise to generalised friezes. Furthermore, we give an explicit combinatorial formula for the entries of these frieze patterns by generalizing the 'T-path formula' of Schiffler which was introduced to give explicit formulas for cluster variables for cluster algebras of type A.

11:00 AM **Sefi Ladkani (Haifa University)**
2-CY-Tilted Algebras for Skew-Symmetrizable Matrices

14:30 PM **Christof Geiß (UNAM)**
Quantum Cluster Algebras and their Specializations

We show that in case a cluster algebra coincides with its upper cluster algebra and the cluster algebra admits a grading with finite dimensional homogeneous components, the corresponding Berenstein-Zelevinsky quantum cluster algebra can be viewed as a flat deformation of the classical cluster algebra.

16:00 PM **Dylan Rupel (Michigan State University)**
Cell Decompositions for Rank Two Quiver Grassmannians

A quiver Grassmannian is a variety parametrizing subrepresentations of a given quiver representation. Reineke has shown that all projective varieties can be realized as quiver Grassmannians. In this talk, I will study a class of smooth projective varieties arising as quiver Grassmannians for (truncated) preprojective representations of an n -Kronecker quiver, i.e. a quiver with two vertices and n parallel arrows between them. The main result I will present is a recursive construction of cell decompositions for these quiver Grassmannians motivated by the theory of rank two cluster algebras. If there is time I will discuss a combinatorial labeling of the cells by which their dimensions may conjecturally be directly computed. This is a report on joint work with Thorsten Weist.

Friday December 14, 2018

9:30 AM **Jianrong Li (Weizmann Institute of Science)**
Geometric Cactus Groups and Geometric Crystals

In this talk (based on joint work with Arkady Berenstein and Jacob Greenstein), we will define a geometric cactus group for each split reductive group. Unipotent bicrystals and geometric crystals were introduced by Arkady Berenstein and David Kazhdan. We will show that geometric cactus groups act on unipotent bicrystals. Therefore they act on geometric crystals. We will show that geometric cactus group actions preserve the potentials on unipotent bicrystals. Moreover, the tropicalizations of certain geometric cactus group actions coincide with the Bender-Knuth moves on Gelfand-Tsetlin patterns studied by Arkady Berenstein and Anatol N. Kirillov.

11:00 AM **Pavel Tumarkin (Durham University)**
Cluster Algebras, Surfaces and Extended Affine Weyl Groups

To any adjacency quiver of a triangulation of an unpunctured surface we can associate a certain group which behaves nicely with respect to mutations. I will discuss a connection between these groups, extended affine Weyl groups, and a conjectural characterization of the adjacency quivers

in terms of positive semidefinite symmetric matrices. The talk is based on joint works with Anna Felikson, John Lawson and Michael Shapiro.

14:30 PM **Bach Nguyen (Temple University)**
Quantum Cluster Algebra at Roots of Unity and Representation Theory

16:00 PM **Gus Schrader (Columbia University)**
Gelfand-Zeitlin Modules under Whittaker Transform

A certain class of infinite dimensional representations of the quantum group $U_q(\mathfrak{sl}(n))$, termed Gelfand-Zeitlin modules, were introduced in the 2000's by Gerasimov, Kharchev, Lebedev and Oblazin. These representations and their generalizations have attracted recent interest due to their role in the theory of quantized K-theoretic Coulomb branch algebras for quiver gauge theories, as constructed by Braverman, Finkelberg and Nakajima. I will explain how the Whittaker transform for the q -difference open Toda chain can be used to reveal the cluster structure behind this class of representations. Joint work with Alexander Shapiro.

Saturday December 15, 2018

9:30 AM **Ba Nguyen (Queen's University)**
Broken Lines and Theta Basis for Type D Cluster Algebras

Lots of research focuses on the combinatorics behind various bases of cluster algebras. This paper studies the natural basis of a type D cluster algebra, which consists of all cluster variables. We introduce a new kind of combinatorial formula for the cluster variables in terms of the so-called globally compatible sequences. Then we give a bijection that relates the formula with the theta functions constructed by Gross, Hacking, Keel and Kontsevich.

11:00 AM **Mikhail Gorsky (Bielefeld University)**
TBA

14:30 PM **Alexander Shapiro (University of Edinburgh)**
On Toda-Ruijsenaars Bispectral Duality

It is well-known that Whittaker functions enjoy a bispectral property — they are simultaneous eigenfunctions for Toda and Ruijsenaars Hamiltonians, where the former act on the coordinate variables while the latter act on the spectral ones. I will show how cluster structure appears in this story and explain a relation to the modular functor conjecture. This is a joint work in progress with Gus Schrader.

16:00 PM **Michael Shapiro (Michigan State University)**
Exotic cluster structures in $\mathbb{C}[GL_n]$

We discuss the progress in the proof of conjecture by M.Gekhtman, A.Vainshtein and M.S. on the existence of cluster structures in the coordinate ring of GL_n compatible with Belavin-Drinfeld R-matrix Poisson-Lie brackets. We proved the conjecture for large class of acyclic BD data.

Conference Organizers

Michael Gekhtman (University of Notre Dame)

Salvatore Stella (University of Haifa)

Alexander Vainshtein (University of Haifa)