Conference on Metric Geometry and Applications

March 14 – 17, 2013
University of Notre Dame

TALK TITLES AND ABSTRACTS
(revised 3-7-13)

All talks held in 127 Hayes-Healy Hall

Stephanie Alexander (University of Illinois – Urbana/Champaign) ***"Warped Products of Metric Spaces"***

We discuss a criterion for a warped product of metric spaces to have an Alexandrov curvature bound, either below (CBB) or above (CBA). Sufficiency was proved earlier, but the necessity of the conditions is new. This topic showcases many tools and constructions of Alexandrov spaces, as well as the dualities between CBB and CBA. (Joint work with Dick Bishop.)

Werner Ballmann (Max Planck Institute for Mathematics) ***"On asymptotic eta-invariants associated to cusps"***

Let M be a complete Riemannian manifold M of finite volume and pinched negative curvature. Let D be a generalized Dirac operator on a Hermitian vector bundle E over M, and assume that D is a Fredholm operator with respect to the space of square integrable sections of E. The topic of the talk is the correction terms for the Atiyah-Patodi-Singer index formula of D related to cusps of M.

Gérard Besson (University of Grenoble) ***"Open three manifolds and Ricci flow"***

We shall describe recent results obtained with L. Bessières and S. Maillot on open 3-manifolds using Ricci flow technique. This is an opportunity to discuss a widely open subject, the geometry of open 3-manifolds.

Dmitri Burago (Pennsylvania State University) ***"A Math Mozaic"***

This won't be a typical conference lecture. Instead I'll give a number of mini-talks on very different topics. The only thing linking the topics together is that they have all been of interest to me in the past several years. A very important part of the lecture will be the presentation of open problems. These will be formulated using only basic material at a level accessible to graduate student.

S-C Chang (National Taiwan University) ***"Pseudo-Einstein Contact Structure via CR Poincare-Lelong Equation"***

In this talk we first shall address the Calabi-Yau-Lee Theorem for pseudo-Einstein contact structure via Poincare-Lelong equation in a closed strictly pseudoconvex CR $(2n+1)$-manifold (M,θ) for $n \geq 2$. We will explain the main technical difficulty via the elliptic method originated by Lee and Cao-Chang, the strategy of this new approach via the parabolic equations. For $n = 1$, it is related to the volume

normalized invariant contact form and has a great implication for CR geometry. A part of talk is the jointed work with Der-Chen Chang and Jingzhi Tie.

Jeff Cheeger (Courant Institute, New York University) ***"Quantitative Behavior of Singular Sets"***

We will discuss joint work with Aaron Naber in which we sharpen Hausdorff estimates on the codimension of the singular set $\mathcal S$ for various geometric nonlinear pde's. Specifically, inside each unit ball, we will estimate the volume of a tube of radius $r\leq 1$ around a set which is larger than $\mathcal S$. At points outside of this tube, the solution has a definite amount of regularity. Namely, at such points, there is a lower bound for what we call the regularity scale.

Christopher Croke (University of Pennsylvania) ***"Scattering and Lens Rigidity"***

We will consider compact Riemannian manifolds M with boundary N. We let IN be the unit vectors to M whose base point is on N and point inwards towards M. Similarly we define OUT. The scattering data (loosely speaking) of a Riemannian manifold with boundary is map from IN to OUT which assigns to each unit vector V of IN a the unit vector W in OUT. W will be the tangent vector to the geodesic determined by V when that geodesic first hits the boundary N again. This may not be defined for all V since the geodesic might be trapped (i.e. never hits the boundary again). A manifold is said to be scattering rigid if any other Riemannian manifold Q with boundary isometric to N and with the same scattering data must be isometric to M. The lens data includes not only the scattering data but also the lengths of the geodesics. In this talk we will discuss some recent results on the scattering (and lens) rigidity problem and related inverse problems. One thing we will discuss is recent work of my graduate student Haomin Weh on the relation between scattering data and lens data for surfaces.

There are a number of manifolds that are known to be lens rigid and there are examples that are not scattering or lens rigid. All of the known examples of non-rigidity have trapped geodesics in them. In particular, we will see that the flat solid torus is scattering rigid. This is the first scattering rigidity result for a manifold that has a trapped geodesic. The main issue is to show that the unit vectors tangent to trapped geodesics in any such Q have measure 0 in the unit tangent bundle of Q. We will also consider scattering rigidity of a number of two dimensional manifolds (joint work with Pilar Herreros) which have trapped geodesics.

Fuquan Fang (University of Notre Dame) ***"Reflection groups, non-negative curvature and Tits geometry"***

A reflection in a euclidean space (sphere) is one of the fundamental notions of symmetry of geometric figures. It plays a central role in Killing and Cartan's work on Lie algebra in 19th century. Reflections groups on a hyperbolic space is important in hyperbolic geometry, and the first example goes back to F. Klein and Poincare. In this talk I will present

(i) A complete classification of reflection groups and the equivariant structures of complete non negatively curved manifolds.

(ii) A complete classification of positively curved polar manifolds of cohomogeneity at least 2, which is achieved partially based on Tits geometry.

(joint works with Karsten Grove and G. Thorbergsson)

Jian Ge (Max Planck Institute for Mathematics) ***"Convex sets and distance function to the boundary."***
In this talk we give some estimations on the concavity of the distance function to the boundary under various convexity assumptions of the boundary. And show some applications.

Nan Li (Pennsylvania State University) ***"Volume Rigidity on Length Spaces"***
In general, a 1-Lipschitz onto, volume preserving map between length metric spaces may not preserve length of paths. We show that such map preserves length of paths if the underlying spaces are Alexandrov spaces (in particular, convex domains in Riemannian manifolds) or the non-collapsed Gromov-Hausdorff limit spaces of manifolds with Ricci curvature uniformly bounded from below. We will also discuss the consequence of these results.

Xiaochun Rong (Rutgers University) ***"Continuity of extremal transitions and flops for Calabi-Yau manifolds"***
We will discuss metric behavior of Ricci-flat Kahler metrics on Calabi-Yau manifolds under algebraic geometric surgeries: extremal transitions or flops. We will prove a version of Candelas and de la Ossa's conjecture: Ricci-flat Calabi-Yau manifolds related via extremal transitions and flops can be connected by a path consisting of continuous families of Ricci-flat Calabi-Yau manifolds and a compact metric space in the Gromov-Hausdorff topology. This is joint work with Yuguang Zhang.

Gang Tian (Princeton University) ***"Einstein Metrics on Fano Manifolds"***
In this talk, I will discuss my recent work on Kahler-Einstein metrics with positive scalar curvature which relates their existence to a geometric stability condition.

Sophia Vassiliadou (Georgetown University) ***"L2-d-bar-cohomology groups of some singular Complex spaces."***
I will discuss results obtained jointly with Nils Ovrelid on the L2-d-bar-cohomology groups of the regular part of some complex spaces with singularities.

Jiaping Wang (University of Minnesota) ***"Geometry of Ricci Solitons"***
As self-similar solutions to the Ricci flows, Ricci solitons are important geometric objects. In this talk, we intend to explain some joint work with Ovidiu Munteanu, which explores the geometry and analysis of gradient Ricci solitons.

Paul Yang (Princeton University) ***"CR Geometry in 3-D"***
In this lecture I will summarize recent work on pseudo-hermitian geometry in 3-D. The analysis involves several conformally covariant operators that have their counterparts in conformal geometry in 4-D, and a new one that does not. In this geometry, the sign of fourth order operator studied by Hirachi and the analogue of the conformal Laplacian plays an important role. Under this sign condition, it is possible to solve the embedding problem and hence to solve the Cauchy-Riemann equations. In addition, the same

sign conditions give the analogue of the positive mass theorem, thus the solution of the CR-Yamabe equation. Finally, a new operator introduced by Branson/Fontana/Morpurgo yields new invariant that can be identified with the renormalized volume, as well as an analogue of the sphere theorem in CR geometry.