NONLINEAR ANALYSIS IN ROME
JUNE 26 – 30, 2017

TITLES AND ABSTRACTS

Minicourses

Luigi Ambrosio (SNS - Pisa)

Title: *Optimal Transport and Ricci curvature*

Abstract: The last few years have witnessed remarkable progress in the study of the synthetic theory of metric measure structures with Ricci lower bounds. This led to new structure, stability and rigidity results, previously known only in the smooth setting or for the so-called Ricci limit spaces. In the lectures I will illustrate the key technical ideas at the basis of these developments, focusing in particular on calculus (gradients, differential, Sobolev functions, Laplacian, etc.) on metric-measure spaces.

Tristan Rivière (ETH - Zurich)

Title: *Minmax Methods in the Calculus of Variations of Curves and Surfaces*

Abstract: The study of the variations of curvature functionals takes its origins in the works of Euler and Bernouilli from the eighteenth century on the Elastica. Since these very early times, special curves and surfaces such as geodesics, minimal surfaces, elastica, Willmore surfaces, etc. have become central objects in mathematics much beyond the field of geometry stricto sensu with applications in analysis, in applied mathematics, in theoretical physics and natural sciences in general. Despite its venerable age the calculus of variations of length, area or curvature functionals for curves and surfaces is still a very active field of research with important developments that took place in the last decades. In this mini-course we shall concentrate on the various minmax constructions of these critical curves and surfaces in euclidian space or closed manifolds.

We will start by recalling the origins of minmax methods for the length functional and present in particular the "curve shortening process" of Birkhoff. We will explain the generalization of Birkhoff's approach to surfaces and the "harmonic map replacement" method by Colding and Minicozzi. We will then recall some fundamental notions of Palais Smale deformation theory in infinite dimensional spaces and apply it to the construction of closed geodesics and Elastica. In the second part of the mini-course we will present a new method based on smoothing arguments combined with Palais Smale deformation theory for performing successful minmax procedures for surfaces. We will present various applications of this so called "viscosity method" such as the problem of computing the cost of the sphere eversion in 3-dimensional Euclidian space.

One-Hour Lectures

Alice Chang (Princeton)

Title: *Compactness of conformally compact Einstein manifolds in dimension 4*

Abstract: Given a class of conformally compact Einstein manifolds with boundary, we are interested to study the compactness of the class under some local and non-local boundary constraints. I will report some joint work with Yuxin Ge and some recent improvements under discussion also with Jie Qing of the problem on the 3+1 setting.

Qing Han (Notre Dame)

Title: *On the Negativity of Ricci Curvatures of Complete Conformal Metrics*

Abstract: A version of the singular Yamabe problem in bounded domains yields complete conformal metrics with negative constant scalar curvatures. In this talk, we study whether these metrics have negative Ricci curvatures. The polyhomogeneous expansions for solutions of the Yamabe equation play an important role in the study.

Paul Laurain (Université de Paris Diderot)

Title: *CMC surfaces in asymptotically flat manifolds*

Abstract: CMC surface plays a very central role in General Relativity. After remembering the state of art, I will discuss some improvement about the uniqueness of such objects. Then I will speak about some generalisations to Willmore surfaces and the notion of quasi-local mass.

Frederic Robert (Université de Lorraine)

Title: *The Hardy-Schrödinger operator with interior singularity: mass and blow-up analysis*

Abstract: We consider the remaining unsettled cases in the problem of existence of positive solutions for the Dirichlet value problem $L_\gamma u - \lambda u = \frac{u^{2^*(s)-1}}{|x|^s}$ on a smooth bounded domain Ω in $\mathbb{R}^n$ ($n \geq 3$) having the singularity 0 in its interior. Here $\gamma < \frac{(n-2)^2}{4}$, $0 \leq s < 2$, $2^*(s) := \frac{2(n-s)}{n-2}$ and $0 \leq \lambda < \lambda_1(L_\gamma)$, the latter being the first eigenvalue of the Hardy-Schrödinger operator $L_\gamma := -\Delta - \frac{\gamma}{|x|^2}$. The higher dimensional case (i.e., when $\gamma \leq \frac{(n-2)^2}{4} - 1$) has been settled sometime ago. In this paper we deal with the case when $\frac{(n-2)^2}{4} - 1 < \gamma < \frac{(n-2)^2}{4}$. If either $s > 0$ or $\{s = 0 \text{ and } \gamma > 0\}$, we show that a solution is guaranteed by the positivity of the "Hardy-singular internal mass" of Ω, a notion that we introduce herein. On the

other hand, the classical positive mass theorem is needed for when $s = 0$, $\gamma \leq 0$ and $n = 3$, which in this case is the critical dimension. We will also discuss some extensions of this work in the nonlocal setting of the fractional Laplacian. This is joint work with Nassif Ghoussoub (UBC, Vancouver).

Jeff Streets (UC - Irvine)

Title: *Generalized Kähler Ricci flow and a generalized Calabi conjecture*

Abstract: Generalized Kähler geometry is a natural extension of Kähler geometry with roots in mathematical physics, and is a particularly rich instance of Hitchin's program of 'generalized geometries.' In this talk I will discuss an extension of Kähler-Ricci flow to this setting. I will formulate a natural Calabi-Yau type conjecture based on Hitchin/Gualtieri's definition of generalized Calabi-Yau equations, then discuss the flow as a tool for resolving this conjecture. The main result is a global existence and convergence result for the flow which yields a partial resolution, and which classifies generalized Kähler structures on hyperKähler backgrounds.

Gabriella Tarantello (Università di Roma "Tor Vergata")

Title: *Liouvilletype systems in the study of non-abelian Chern-Simons vortices*

Abstract: We discuss elliptic systems of Liouville type in presence of singular sources, arising from the study of non-abelian (selfdual) Chern-Simons vortices. We shall focus on the search of the so-called non-topological configurations and present some recent results as well as (still many) open questions.

Susanna Terracini (Università di Torino)

Title: *Regularity of the optimal sets for spectral functionals, Part I: sum of eigenvalues*

Abstract: In this talk we deal with the regularity of optimal sets for a shape optimization problem involving a combination of eigenvalues, under a fixed volume constraints. As a model problem, consider

$$\min\Big\{\lambda_1(\Omega)+\cdots+\lambda_k(\Omega)\ :\ \Omega\subset\mathbb{R}^d,\ \text{open}\ ,\ |\Omega|=1\Big\},$$

where $\lambda_i(\cdot)$ denotes the eigenvalues of the Dirichlet Laplacian and $|\cdot|$ the d-dimensional Lebesgue measure. We prove that any minimizer Ω_{opt} has a regular part of the topological boundary which is relatively open and C^∞ and that the singular part has Hausdorff dimension smaller than $d - d^*$, where $d^* \geq 5$ is the minimal dimension allowing the existence of minimal conic solutions to the bow-up problem.

We mainly use techniques from the theory of free boundary problems, which have to be properly extended to the case of vector-valued functions: nondegeneracy property, Weiss-like monotonicity formulas with area term; finally through the

properties of non tangentially accessible domains we shall be in a position to exploit the "viscosity" approach recently proposed by De Silva.

This is a joint work with Dario Mazzoleni and Bozhidar Velichkov.

Juncheng Wei (UBC)

Title: *Finite Morse Index Implies Finite Ends*

Abstract: We prove that finite Morse index solutions to the Allen-Cahn equation in R^2 have finitely many ends and linear energy growth. The main tool is a curvature decay estimate on level sets of these finite Morse index solutions, which in turn is reduced to a problem on the uniform second order regularity of clustering interfaces for the singularly perturbed Allen-Cahn equation. Using an indirect blow-up technique, inspired by the classical Colding-Minicozzi theory in minimal surfaces, we show that the obstruction to the uniform second order regularity of clustering interfaces in R^n is associated to the existence of nontrivial entire solutions to a (finite or infinite) Toda system in R^{n-1}. For finite Morse index solutions in R^2, we show that this obstruction does not exist by using information on stable solutions of the Toda system. (Joint work with Kelei Wang)

Short Communications

Giovanni Catino (Politecnico di Milano)

Title: *Einstein metrics and Ricci solitons*

Abstract: In this talk I will review some classification and rigidity results for Einstein metrics and Ricci solitons. In particular, I will present a new Bochner-type identity on four-dimensional Einstein manifolds and a very recent uniqueness result for compact shrinking Ricci solitons in dimension four.

Gyula Csató (Universidad de Concepción)

Title: *About Hardy-Sobolev, Moser-Trudinger and isoperimetric inequalities with weights*

Abstract: The well known Sobolev embedding states that if $1 < p < n$, then there exists a constant $C > 0$ such that

$$\|u\|_{L^{p^*}(\mathbb{R}^n)} \leq C\|Du\|_{L^p(\mathbb{R}^n)} \quad \forall\, u \in C_c^\infty(\mathbb{R}^n), \qquad p^* = \frac{np}{n-p}.$$

In the limiting case $p = n$, the corresponding result in a bounded domain Ω is the Moser-Trudinger embedding and it can be stated as follows (here for simplicity in dimension 2):

$$\sup_{u\in C_c^\infty(\Omega),\ \|Du\|_{L^2}\leq 1} \int_\Omega e^{\alpha|u|^2} \leq C(\Omega), \quad 0 \leq \alpha \leq 4\pi$$

for some constant $C = C(\Omega)$. These embeddings have many weighted versions, for instance one of them is

$$\left\| \frac{u}{|x|^\tau} \right\|_{L^{p^*}(\mathbb{R}^n)} \leq C\|Du\|_{L^p(\mathbb{R}^n)} \quad \forall u \in C_c^\infty(\mathbb{R}^n), \qquad 0 \leq \tau \leq 1, \quad p^* = \frac{np}{n-p+p\tau}.$$

There are many more types of weighted versions and in general these are called Hardy-Sobolev or Caffarelli-Kohn-Nirenberg inequalities. There is also a corresponding weighted version for the Moser-Trudinger embedding, called the singular Moser-Trudinger embedding. First I will give a short overview on these embeddings, on best constants and extremal functions. I will mention briefly some of their geometric versions, called isoperimetric inequalities with densities, respectively some recent results on the subject. Then I will mainly concentrate on the Moser-Trudinger and singular Moser-Trudinger embeddings, and on the existence of extremal functions. I will explain a connection between the harmonic transplantation method of Flucher and an isoperimetric problem with density. This is joint work with Prosenjit Roy.

Alessandro Iacopetti (Università di Torino)

Title: *Radial graphs of prescribed mean curvature in the Lorentz-Minkowski space*

Abstract: A radial graph is characterized, from a geometric point of view, as a hypersurface $M \subset \mathbb{R}^{n+1}$ that is star-shaped relative to the origin, that is, each ray emanating from the origin intersects M once at most. In this talk we show some recent results concerning the problem finding radial graphs of prescribed mean curvature in the Lorentz-Minkowski space. We will discuss also the issue of their uniqueness and we will compare our results with those in the Euclidean framework.

These results are contained in a work in collaboration with D. Bonheure (Université Libre de Bruxelles).

Casey Kelleher (UC - Irvine)

Title: *TBA*
Abstract:

Jae Min Lee (CUNY Graduate Center)

Title: *Local Well-posedness of the Camassa-Holm equation on the real line*

Abstract: In this talk we will prove the local well-posedness of the Camassa-Holm equation on the real line in the space of continuously differentiable diffeomorphisms with an appropriate decaying condition. This work was motivated by G. Misiolek who proved the same result for the Camassa-Holm equation on the periodic domain. We use the Lagrangian approach and rewrite the equation as an ODE on the Banach manifold. Then by using the standard ODE technique, we prove existence and uniqueness. Finally, we show the continuous depdendence of the solution on the initial data by using the topological group property of the diffeomorphism group.

Stephen McKeown (Princeton)

Title: *Cornered Asymptotically Hyperbolic Metrics*

Abstract: I will present results on cornered asymptotically hyperbolic spaces, which have a finite boundary in addition to the usual infinite boundary. After introducing the setting, I will present a normal form near the corner for these spaces. Using this, I will then discuss formal existence and uniqueness, near the corner, of asymptotically hyperbolic Einstein metrics, with a CMC-umbilic condition imposed on the finite boundary. This is doctoral work under C. Robin Graham.

Francesca De Marchis (La Sapienza Università di Roma)

Title: *Asymptotic behavior of solutions to Lane-Emden problems*

Abstract: We will consider the semilinear Lane-Emden problem with Dirichlet boundary conditions in a bounded domain of $\mathbb{R}^2$ and we will analyze the asymptotic behavior of sequences of solutions as the exponent p of the nonlinearity tends to infinity. We will discuss different results obtained in collaboration with M. Grossi, I. Ianni and F. Pacella.

Mariana Smit Vega Garcia (University of Washington)

Title: *The singular free boundary in the Signorini problem*

Abstract: In this talk I will overview the Signorini problem for a divergence form elliptic operator with Lipschitz coefficients, and I will describe a few methods used to tackle two fundamental questions: what is the optimal regularity of the solution, and what can be said about the singular free boundary in the case of zero thin obstacle. The proofs are based on Weiss and Monneau type monotonicity formulas. This is joint work with Nicola Garofalo and Arshak Petrosyan.

Michael Smith (UC - Berkeley)

Title: *Local $L^{\frac{1}{2}-\epsilon}$ bound on Ricci curvature assuming a Ricci curvature bound from below*

Abstract: I will discuss the following result: Given a Riemannian manifold with Ricci curvature uniformly bounded from below, we show that, for $p < 1/2$, the L^p -norm of the Ricci curvature over a ball of a given radius is bounded from above. To be precise,

Proposition 0.1. *let M^n be a complete Riemannian manifold and $p_0 \in M$. Assume the Ricci curvature satisfies $Ric \geq -1$ on $B(p_0, 5)$. Then for any $0 < \epsilon < 1/2$,*

$$||Ric||_{B(p_0,1),L^{1/2-\epsilon}} \leq C(\epsilon)Vol(B(p_0,5))^{2\epsilon}.$$

Azahara de la Torre Pedraza (Basel)

Title: *Gluing methods for the fractional Yamabe problem with isolated singularities*

Abstract: We construct some solutions for the fractional Yamabe problem with isolated singularities, problem which arises in conformal geometry, $(-\Delta)^\gamma u = c_{n,\gamma} u^{\frac{n+2\gamma}{n-2\gamma}}$, $u > 0$ in $\mathbb{R}^n \setminus \Sigma$. The fractional curvature, a generalization of the usual scalar curvature, is defined from the conformal fractional Laplacian, which is a non-local operator constructed on the conformal infinity of a conformally compact Einstein manifold.

When the singular set Σ is composed by one point, some new tools for fractional order ODE can be applied to show that a generalization of the usual Delaunay solves the fractional Yamabe problem with an isolated singularity at Σ.

When the set Σ is a finite number of points, using gluing methods, we will provide a solution for the fractional Yamabe problem with singularities at Σ. In order to preserve the non-locality of the problem, we need to glue infinitely many bubbles per point removed. This seems to be the first time that a gluing method is successfully applied to a non-local problem.

This is a joint work with Weiwei Ao, María del Mar González and Juncheng Wei.