BOOK OF ABSTRACTS

68^{th} Midwest PDE Seminar
November $4-6$, 2011
University of Notre Dame

Conference Organizers:
Qing Han (qhan@nd.edu),
Alex Himonas (himonas@nd.edu),
Bei Hu (b1hu@nd.edu),
Gerard Misiolek (gmisiole@nd.edu)

Sponsors: *The 68^{th} Midwest Partial Differential Equations Seminar is partially funded by the National Science Foundation, the Graduate School, the College of Science, the Department of Mathematics, the Department of ACMS and the Center for Mathematics of the University of Notre Dame.*

Special thanks for assisting the organization: Melissa Davidson, Katie Grayshan, John Holmes, David Karapetyan, Ryan Thompson, and Lisa Tranberg.

Jerry Bona (University of Illinois, Chicago)
"Comparisons Between Model Equations for Water Waves"
The discussion will center around the derivation of various systems of partial differential equations that formally provide models for water wave motion. Of special interest will be rigorous comparisons between some of these models.

Hongqiu Chen (University of Memphis)
"Analysis on Stability of Solitary-Wave Solutions for a System of Nonlinear Dispersive Equations"
Consider the system

$$\begin{cases} u_t + u_{xxx} + P(u,v)_x = 0, \\ v_t + v_{xxx} + Q(u,v)_x = 0 \end{cases}$$

of coupled KdV-equations introduced by Bona, Cohen and Wang, where $u = u(x,t), v = v(x,t)$ are functions defined on $\mathbb{R} \times \mathbb{R}^+$, $P(u,v) = Au^2 + Buv + Cv^2$ and $Q(u,v) = Du^2 + Euv + Fv^2$ in which $A, B, \cdots, F$ are real number constants. There are up-to three explicit solitary-wave solutions of hyperbolic square functions. Moreover, if system of linear equations

$$\begin{cases} 2Ba + (E - 2A)b - 4Dc = 0, \\ 4Ca + (2F - B)b - 2Ec = 0 \end{cases}$$

has solutions (a, b, c) such that $4ac > b^2$, then, the stability and instability of the solitary-wave solutions can be checked by a simple and straightforward algebraic function.

Ko-Shin Chen (Indiana University - Bloomington)
"Vortex Annihilation for Ginzburg-Landau on a Manifold"
We investigate the Ginzburg-Landau heat flow posed on certain surfaces of revolution with boundary. First we derive the parabolic Pohozaev identity for heat flow on a manifold. Then we use it to prove that after a finite time T, the solution u to the PDE has no vortices.

Xinfu Chen (University of Pittsburgh)
"Effects of White Noise in Multistable Dynamics"
We study the invariant measure of multistable dynamics under the influence of white noise. We show that the invariant measure exists and in the limit of vanishing white noise, the invariant measure approaches a Dirac type measure concentrated at the most stable equilibria if fluctuations are uniform; however, a lesser stable equilibrium may be selected by the fluctuation if its ability to fluctuate is sufficiently smaller than other stable equilibria. Certain related mathematical issues are also addressed.

James Colliander (University of Toronto)
"Interaction Morawetz Estimate for Gauged Schrödinger"
This talk describes an interaction Morawetz estimate for the magnetic Schrödinger equation under certain smallness conditions on the gauge potentials. This talk describes collaborative work with Magdalena Czubak (SUNY Binghamton) and Jeonghun Lee (Minnesota).

Daniel da Silva (University of Rochester)
"On the Regularity of the 2+1 Dimensional Skyrme Model"
One of the most interesting open problems concerning the Skyrme model of nuclear physics is the regularity of its solutions. In this talk, we will discuss the non-concentration of energy for the 2+1 dimensional Skyrme model, which is an important step towards a global regularity theory.

Melissa Davidson (University of Notre Dame)
"The Generalized Reduced Ostrovsky Equation and its Continuity Properties"
It is shown that the data-to-solution map for the generalized Reduced Ostrovsky (gRO) equation is not uniformly continuous on bounded sets in Sobolev spaces on the circle with exponent $s > 3/2$. Considering that for this range of exponents the gRO equation is well-posed with continuous dependence on initial data, this result makes the continuity of the solution map an optimal property. However, if a weaker H^r-topology is used then it is shown that the solution map becomes Hölder continuous in H^s.

Marcelo Mendes Disconzi (Stony Brook University)
"A Compactness Theorem for the Yamabe Problem on Manifolds with Boundary"
We study the problem of conformal deformation of Riemannian structure to constant scalar curvature with zero mean curvature on the boundary. We prove compactness for the full set of solutions when the boundary is umbilic and the dimension $n \leq 24$. The Weyl Vanishing Theorem is also established under these hypotheses.

Dan Geba (University of Rochester)
"The Global Regularity Problem for Several Classical Field Theories"
In this lecture, we plan to discuss first the significant progress made, especially in the last twenty years, in the mathematical understanding of the nonlinear sigma model (i.e., the wave maps system) of nuclear physics. We will then focus our attention on a set of important field theories which generalize this system (e.g., Adkins-Nappi, Skyrme, and Fadeev models) and describe recent progress, together with open questions, in the direction of global regularity. This is in part joint work with Kenji Nakanishi, Sarada Rajeev, and Daniel da Silva.

Katie Grayshan (University of Notre Dame)
"Nonuniform Dependence for the Cauchy Problem of the Periodic b-family Equation"
For Sobolev exponents $s > 3/2$, the Cauchy problem for the periodic b-family equation has been shown to be well-posed (in the sense of Hadamard) in $H^s(\mathbb{T})$. In particular, the associated solution map is continuous as a map from $H^s(\mathbb{T})$ into $C([0,T]; H^s(\mathbb{T}))$. We prove that this map is not uniformly continuous using approximate solutions together with delicate commutator and multiplier estimates. The novelty of the proof lies in the fact that it makes no use of conserved quantities.

William Green (Eastern Illinois University)
"Dispersive Estimates for Schrödinger Operators in Dimension Two with Obstructions at Zero Energy"
We investigate $L^1(\mathbb{R}^2) \to L^\infty(\mathbb{R}^2)$ dispersive estimates for the two-dimensional Schrödinger operator $H = -\Delta + V$ when there are obstructions, resonances or an eigenvalue, at zero energy. In particular, with sufficient decay of the potential V, we show that the existence of an s-wave resonance at zero energy does not destroy the t^{-1} decay rate. We also show that the existence of a p-wave resonance or an eigenvalue at zero energy leads to a bounded evolution.

Curtis Holliman (University of Alabama, Birmingham)
"Norm Inflation and Ill-posedness for the Degasperis-Procesi Equation"
For $s < 3/2$ it is shown that the Cauchy problem for the Degasperis-Procesi equation (DP) is ill-posed in Sobolev spaces H^s.

When $1/2 \le s < 3/2$ then ill-posedness is a result of norm inflation. This means that there are DP solutions with arbitrarily small initial data, as measured in the H^s-norm, that grow to an inversely proportional size in an arbitrarily small time in the solution space.

For the same family of solutions, we see that in the case $s < 1/2$, ill-posedness is caused by shrinking to zero lifespans. Using this result in conjunction with scaling laws for the DP equation, these families of solutions are then shown violate conditions for continuity of the data-to-solution map. This work is in collaboration with Alex Himonas.

John Holmes (University of Notre Dame)
"Regularity Issues of the Generalized Burgers Equation"
We analyze the regularity properties for the generalized Burgers' equation,

$$u_t - u_{xx} + u^k u_x = f.$$

We show that given analytic initial data, the solution will be analytic in space and Gevrey-2 in time. We will further show that this is optimal by construction of explicit solutions which fail to be analytic in time.

David Karapetyan (University of Notre Dame)
"On the Hölder Continuity of the Data to Solution Map for the Hyperelastic Rod Equation"
It is proved that for $s > 3/2$, $r < s$ the data-to-solution map for the hyperelastic rod (HR) equation is Hölder in H^s with index $\gamma = \gamma(r)$ if the norm is replaced with a weaker H^r norm. Furthermore, the result is optimal, in the sense that it does not hold for $\gamma + \epsilon$, for any $\epsilon > 0$.

Marcus Khuri (SUNY, Stony Brook)
"The Jang Equation of General Relativity and Applications"
In their proof of the Positive Mass Theorem of General Relativity, Schoen and Yau employed a quasi-linear elliptic equation introduced by the physicist P. S. Jang. In this talk, we describe a generalized version of this equation which is calibrated to the behavior of static spacetimes. We will analyze its blow-up behavior and describe applications which yield lower bounds for the ADM mass.

This is joint work with Qing Han.

Nguyen Lam (Wayne State University)
"Existence and Multiplicity of Solutions to Equations of $N-$Laplacian Type with Critical Exponential Growth in $\mathbb{R}^N$"
In this talk, we deal with the existence of solutions to the nonuniformly elliptic equation of N-Laplacian type where the nonlinearity behaves like $\exp(|u|^N/(N-1))$. Also, using the minimization and the Ekeland variational principle, we obtain multiplicity of weak solutions to the perturbation equation. Finally, we will prove the above results when our nonlinearity f doesn't satisfy the well-known Ambrosetti-Rabinowitz condition and thus derive the existence and multiplicity of solutions for a wider class of nonlinear terms f.

Matthew Masarik (University of Michigan)
"The Wave Equation in General Black Hole and Particle-like Geometries"
We discuss the decay of solutions to the linear wave equation in general black hole and particle-like (singularity free, asymptotically flat) geometries. In both cases we provide criteria that are easy to check which guarantee decay of the linear wave equation on these spacetimes.

Nathan Pennington (Kansas State University)
"Global Solutions to the Lagrangian Averaged Navier-Stokes Equation in $B^{n/p}_{p,q}(\mathbb{R}^n)$"
The Lagrangian Averaged Navier-Stokes equations are a recently derived approximation to the Navier-Stokes equations. As the name suggests, the Lagrangian Averaged Navier-Stokes are derived by averaging at the Lagrangian level, and the resulting PDE has more easily controlled long time behavior than the Navier-Stokes equations. Previously, the existence of global solutions has been proven for initial data in the Sobolev space $u_0 \in H^{3/4,2}(\mathbb{R}^3)$ and, separately, the Besov space $u_0 \in B^{n/2}_{2,q}(\mathbb{R}^n)$. In this paper, we use a non-standard, interpolative method to prove the existence of global solutions to the equation with initial data $u_0 \in B^{n/p}_{p,q}(\mathbb{R}^n)$ for $3 \leq n < 6$ for any $p > n$.

Gustavo Ponce (University of California Santa Barbara)
"The IVP for the Benjamin-Ono Equation in Weighted Sobolev Spaces"
(Joint work with G. Fonseca and F. Linares) We study the initial value problem associated to the Benjamin-Ono equation. The aim is to establish persistence properties of the solution in the weighted Sobolev spaces.

We also prove some unique continuation properties of solutions to this equation. In particular, we shall establish optimal decay rate for the solutions of this model.

Finally, we shall compare our results with those previously proven for other dispersive models, including the generalized Korteweg-de Vries equation, the Camassa-Holm equation, and the Schrödinger equation.

Wilhelm Schlag (University of Chicago)
"Invariant Manifolds and Hamiltonian Evolution Equations"
The field of nonlinear dispersive evolutions equations has undergone rapid changes in recent years. The equations in question are Hamiltonian and encompass a wide class ranging from nonlinear Klein-Gordon, wave, and Schrödinger equations on the one hand, to more geometric

equations such as wave maps and other so-called field equations of physics on the other hand.

These equations have traditionally been studied from the point of view of the fundamental well-posedness problem locally in time which often requires large amounts of analytical machinery. The question of global in time properties of the evolution is the subject of much ongoing research in nonlinear evolution equations. Within the past five to six years, several open problems have been settled in the field by introducing new ideas such as concentration-compactness for evolution equations, and the use of invariant manifolds from hyperbolic dynamics. We shall give an overview over some of these developments.

Joel Smoller (University of Michigan)
"Gravitation"
I will begin by discussing gravitation, beginning with Newton, and culminating with Einstein's theory of General Relativity. I will then consider some recent results based upon GR: black holes, big-bang cosmology, and the so called anomalous acceleration of the universe.

Peter Sternberg (Indiana University)
"Vortex Motion on a Closed Surface"
I will discuss solutions with vortices to the Ginzburg-Landau heat flow and to the Gross-Pitaevskii system, both posed on a closed surface. In both cases, the focus is on making connections between these flows and the limiting flow of point vortices as the parameter controlling the size of the vortex core tends to be zero.

This talk covers joint work with Ko-Shin Chen and Michael Gelantalis.

Anthony Suen (Indiana University - Bloomington)
"Global Weak Solutions of the Equations of 3-D Compressible Magnetohydrodynamics"
We prove the global-in-time existence of weak solutions of the equations of compressible magnetohydrodynamics in three space dimensions with initial data small in L^2 and initial density positive and essentially bounded. A great deal of information concerning partial regularity is obtained: velocity, vorticity, and magnetic field become relative smooth in positive time (H^1 but not H^2) and singularities in the pressure cancel those in a certain multiple of the divergence of the velocity, thus giving concrete expression to conclusions obtained formally from the Rankine-Hugoniot conditions.

Feride Tiglay (Fields Institute, Canada)
"Integrable Evolution Equations on Spaces of Tensor Densities"
In a pioneering paper V. Arnold presented a general framework within which it is possible to employ geometric and Lie theoretic techniques to study the equations of motion of a rigid body in $\mathbb{R}^3$ and the equations of ideal hydrodynamics. I will describe how to extend his formalism and introduce two integrable PDE. One of the equations turns out to be closely related to the inviscid Burgers equation while the other has not been identified in any form before. These two PDE possess all the hallmarks of integrability: the existence of a Lax pair formulation, a bi-Hamiltonian structure, the presence of an infinite family of conserved quantities and the ability to write down explicitly some of its solutions.

I will present some results concerning the well-posedness of the corresponding Cauchy problem and the peakon solutions.

Ihsan Topaloglu (Indiana University)

"A Nonlocal Isoperimetric Problem on the Two-sphere"

There is currently much interest on the mathematical analysis of phase separation of block copolymers and their sharp interface limit leading to a nonlocal isoperimetric problem (NLIP). In this talk I will analyze the NLIP on the two-sphere and characterize the global minimizer when the parameter controlling the influence of the nonlocality is small. Furthermore, I will demonstrate stability/instability results of certain critical points depending on where in the parameter regime one looks.

Kazuo Yamazaki (Oklahoma State University)

"On the Global Regularity of Generalized Leray-alpha Type Models"

We generalize Leray-alpha type models studied in Cheskidov et al (2005 Proc. R. Soc. A 461, 629-649) and Linshiz et al (2007 Journal of Mathematical Physics, 48, 065504) via fractional Laplacians and employ Besov space techniques to obtain global regularity results with the logarithmically supercritical dissipation.

Matthias Youngs (Indiana University - Bloomington)

"Existence of Weak Solutions to a Model for Sparse, One-dimensional, Non-barotropic Fluid"

We prove the global-in-time existence of weak solutions to a model for a one-dimensional, viscous, compressible, non-barotropic fluid initially occupying a general open subset of finite length. The fluid equations are applied only on the support of the density, understood in the sense of distributions. This support must be tracked in time and accommodations must be made for possibly infinitely many collisions of fluid packets occurring on a possibly dense set of collision times. Our approach avoids certain nonphysical properties of solutions which are constructed as limits of solutions in which artificial mass has been inserted and, instead, gives a solution that is locally momentum conserving. We build and prove a reasonable theory for finitely many fluid packets and then use weak compactness to let the number of fluids go to infinity.