Quivers and Bipartite Graphs: Physics and Mathematics
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ABSTRACTS

Wilson Loop diagrams and Grassmannian Geometry

Susama Agarwala, University of Nottingham

In this talk I discuss Wilson loop diagrams, a way of analyzing particle interactions in SYM N=4 theory, both in their standard form and as bipartite graphs. I show how these diagrams represent cells of a Grassmannian. I discuss how physical requirements imposed upon the diagram for completely non-geometric reasons correspond to geometric properties of the cells.

The duality conjectures for quantum cluster varieties from surfaces

Dylan Allegretti, Yale University

A well known result of Fock and Goncharov states that the algebra of regular functions on a cluster variety associated to a surface possesses a canonical vector space basis parametrized by points of a dual moduli space. This algebra of functions can be canonically quantized, and Fock and Goncharov conjectured that their canonical basis could be deformed to a canonical set of elements in the quantized algebra. In this talk, I will discuss my recent work proving Fock and Goncharov's conjectures for quantum cluster varieties associated to surfaces. The results are partly based on joint work with Hyun Kyu Kim.

Nonnoetherian Dimer Algebras and Noncommutative Crepant Resolutions

Charlie Beil, University of Bristol

It is well known that every cancellative (i.e., consistent) dimer algebra is a noncommutative crepant resolution (NCCR), and every 3-dimensional affine toric Gorenstein singularity admits an NCCR given by a cancellative dimer algebra. However, dimer algebras which are cancellative are quite rare, and we consider the question: how close are nonnoetherian (homotopy) dimer algebras to being NCCRs? To address this question, I will propose a generalization of NCCRs to nonnoetherian tiled matrix rings. I will then describe a class of dimer algebras which are nonnoetherian NCCRs.

Snake Graphs and Continued Fractions

Ilke Canakci, University of Leicester

Snake graphs are planar graphs appeared first in the context of cluster algebras associated to marked surfaces. In their first incarnation, snake graphs were used to give formulas for generators of cluster algebras. Along with further investigations and several applications, snake graphs were also studied from a more abstract point of view as combinatorial objects. This talk will report on their link to continued fractions inspired by planar graphs associated to Markov numbers.

Stratification of quiver Grassmannians via Theta Bases

Man Wai Cheung, University of California – San Diego

Scattering diagrams were first developed by Kontsevich and Soibelman, later by Gross and Siebert, to solve problems in mirror symmetry. Later on, it is found that the diagrams encodes information about cluster translation. While the canonical basis for cluster algebra is still unclear, Gross-Hacking-Keel-Kontservich proposed one, the theta basis, for cluster algebra constructed from scattering diagram. And the construction is able to solve one of the major conjecture in cluster algebra. In the talk, we are going to describe this construction. Furthermore, we will discuss how the broken lines on scattering diagram give a stratification of quiver Grassmannians using this setting.

Discrete Symmetries, Framed BPS States, and Quantum Monodromies of BPS Lines

Michele del Zotto, Harvard University

In presence of a BPS line defect, the spectrum of excitations of a 4d N=2 theory develops a novel sector given by those BPS states which are bound to the defect. These states are categorified using framed representation theory. We introduce and discuss mutations of framed BPS quivers and their relation to the physics of (anti)wall-crossings of framed BPS particles. This is deeply related to the theory of cluster algebras and provides a novel route to the categorification of cluster variables.

Exotic Generalized Cluster Structures

Idan Eisner, Technion – Israel Institute of Technology

Using the notion of compatibility between cluster structures and Poisson brackets, Gekhtman, Shapiro and Vainshtein studied the connection between the two in the ring of regular functions on a simple complex Lie group G. Using the Belavin-Drinfeld classification of Poisson-Lie groups, they conjectured that for a given Poisson bracket on O(G) there exists a compatible cluster structure, whose rank and other properties can be determined from the initial Belavin-Drinfeld data. This conjecture was verified for a number of classes on G=SL_{n}, but not much is known for other Lie groups. We show that for SP_{6} the standard case satisfies the conjecture, but the exotic case gives rise to a generalized cluster structure.

Cluster Algebras of Rank 3

Anna Felikson, Durham University

We present a geometric realization for all cluster algebras of rank 3. This realization is via linear reflection groups for acyclic mutation classes and via groups generated by π-rotations for the cyclic ones. The geometric behavior of the model turns out to be controlled by the Markov's constant $p^2+q^2+r^2-pqr$, where p,q,r are the elements of the exchange matrix. We also consider non-integer exchange matrices and find the classification of skew-symmetric mutation-finite real matrices of order 3. This is a joint work with Pavel Tumarkin.

Webs, clusters, and Grassmannians

Chris Fraser, University of Michigan

Fomin and Pylyavskyy have proposed a description of the cluster combinatorics for Grassmannians of three-dimensional subspaces in terms of tensor diagrams and Kuperberg's web basis. I will introduce this combinatorics, focusing on concrete example calculations. We will show that the cluster structures on these Grassmannians are related to cluster structures on Fock-Goncharov spaces of configurations of affine flags, via a new notion called a quasi-isomorphism. The relationship between these spaces can be used to provide evidence for the Fomin-Pylyavskyy conjectures.

Exchange Graphs of Polygonal Subdivisions

Alexander Garver, University of Minnesota

Exchange graphs of surface triangulations appear naturally in the study of cluster algebras. These exchange graphs admit a natural acyclic orientation called the oriented exchange graph by Brustle, Dupont, and Pérotin. The acyclic orientation is determined by selecting a triangulation, which will be the unique source of the oriented exchange graph. Oriented exchange graphs have been useful for studying maximal green sequences and they are isomorphic to many important posets of representation theoretic objects. We introduce oriented exchange graphs of polygonal subdivisions (equivalently, partial triangulations) of the disk, which we refer to as oriented flip graphs. The structure of the oriented flip graph of a polygonal subdivision can be understood using the representation theory of a representation-finite gentle algebra defined by the polyogonal subdivision. This algebra has been called a tiling algebra by Simoes and Parsons. We show that an oriented flip graph is isomorphic to the poset of torsion pairs of the associated tiling algebra and give a combinatorial description of all 2-term simple-minded collections in the bounded derived category of the tiling algebra. This is joint work with Thomas McConville.

Cluster variables and perfect matchings

Alastair King, University of Bath

Certain cluster variables (the twists of Plucker coordinates) in the homogeneous coordinate rings of Grassmannians have been expressed by Marsh & Scott as dimer partition functions, i.e. as a sum over perfect matchings. Their formula is inspired by the snake graph formula of Canakci & Schiffler, which also gives a special case. The aim of the talk is to make a link with the famous Caldero-Chapoton formula for cluster variables, by interpreting perfect matchings as modules. This is work in progress with I. Canakci and M. Pressland.

Internally Calabi-Yau algebras

Matthew Pressland, MPIM Bonn

To any dimer model on a closed surface, one can associate a Jacobian algebra, which, by work of Broomhead, is 3-Calabi-Yau if the dimer model is consistent. A modification of the construction associates a frozen Jacobian algebra to a dimer model on a surface with boundary. In this context it is reasonable to ask when this algebra is Calabi-Yau 'away from the boundary', in a sense I will make precise with the definition of an internally Calabi-Yau algebra. Such algebras play an important role in the categorification programme for cluster algebras, and the main goal of the talk will be to explain this connection.

Permutation Centralizer Algebras, Underlying Polynomial Invariants, and Correlators

Sanjaye Ramgoolam, Queen Mary University London

I describe a class of permutation centralizer algebras which underlie the combinatorics of gauge invariant polynomials in matrix or tensor variables. One family of such algebras, closely related to Littlewood-Richardson coefficients, organises the counting and correlators of 2-matrix invariants. The structure of the algebras contain precise information about the enhanced symmetry charges which distinguish the invariants. Another family, closely related to Kronecker product coefficients, organises the counting and correlators of 3-index tensor invariants. Talk will be based on http://arxiv.org/abs/1601.06086

Dessins d'Enfants in N=2 Generalised Quiver Theories

James Read, University of Oxford

I discuss one context in which Grothendieck's \emph{dessins d'enfants} arise in the $\mathcal{N}=2$ generalised quiver theories recently considered by Gaiotto. Specifically, they are so-called ribbon graphs at certain isolated points in the moduli spaces of such theories. I consider implications of this observation for our understanding of these theories, and highlight connections to other work on bipartite graphs, quiver theories, and the modular group.

Polytopes and Mirror Symmetry for Grassmannians

Konstanze Rietsch, King’s College London

Together with Robert Marsh we wrote down a mirror 'superpotential' W_q for a general Grassmannian and showed that W_q encodes the quantum cohomology of the Grassmannian and related structures. The superpotential is a function defined on a certain cluster variety lying inside another (Langlands dual) Grassmannian. In this talk I will report on joint work with Lauren Williams which gives a different perspective on the same superpotential. In this work we show that the cluster tori lying in the cluster variety are in a natural sense dual to certain `network' tori in the original Grassmannian. We use these two families of tori to give two very different constructions of a set of polytopes whose integral points paramterize bases of representations of GL_n with highest weight a multiple of a fundamental weight.

Dual Canonical Basis Conjecture via Categorifications of Cluster Algebras and Quantum Groups

Dylan Rupel, University of Notre Dame

At the heart of the definition of a quantum cluster algebra is a deep conjecture that the recursively, combinatorially defined cluster monomials of a conjectural quantum cluster algebra structure are elements of the (dual) canonical basis of a quantum group. In this talk I will describe a proof of this "dual canonical basis conjecture" in the special case of acyclic, skew-symmetric quantum cluster algebras by relating the categorifications on either side of this picture.

Cluster Algebras and Chern-Simons Invariants

Ralf Schiffler, University of Connecticut

Hikami and Inoue have established a relation between the complex volume of knots and a particular kind of cluster algebras. The computation of Chern-Simons invariants can be reduced to solving a system of equations in the cluster algebra. The cluster algebras that appear in this context are those associated to a disk with m punctures and 4 marked points on the boundary.

In this talk I will explain how to construct the system of equations and then discuss how one can try to solve it in the cluster algebra using combinatorial formulas in terms of perfect matchings of snake graphs.

Upper Cluster Algebras Revised

Michael Shapiro, Michigan State University

In the commonly accepted definition of the upper cluster algebras, the stable variables are invertible in the ring of coefficients. In particular, these assumptions were used in the proof of the statement that the upper bounds do not depend on the seed if the exchange matrix has full rank by Berenstein, Fomin and Zelevinsky.

We showed that one can replace the ring of coefficients by the ring of the regular polynomials of the stable variables, and independence of the upper bounds still holds. This is joint work with M. Gekhtman and A. Vainshtein.

Cluster Algebras via Reflection Groups

Pavel Tumarkin, Durham University

I will describe a construction of a geometric model for all acyclic cluster algebras. The construction is based on the geometry of reflection groups acting in quadratic spaces. The talk is based on a joint work with Anna Felikson.

Cluster Algebraic Interpretation of Infinite Frieze

Hannah Vogel, Durham University

We construct a periodic infinite friezes using a class of elements of a cluster algebra of type D_n. This interpretation can be used to explain a n-arithmetic property studied by Tschabold, and growth behavior studied by Baur, Fellner, Parsons, and Tschabold, along with other nice symmetries. We also give a formula in terms of Conway-Coxeter and Broline-Crowe-Isaacs's matching numbers of a triangulation of a once-punctured disk for the Laurent polynomial expansion of any cluster variable. This interpretation generalizes to type D the well known correspondence between CC&BCI's matching numbers and various combinatorial interpretations of cluster variables of type A. This is joint work with Emily Gunawan and Gregg Musiker.

Clusters are Exact Lagrangian Surfaces

Harold Williams, University of Texas - Austin

We explain how many appearances of cluster structures in mathematics and physics can be understood as follows: 1) Lots of important geometric objects (moduli spaces of local systems, positroid strata, various integrable systems) are spaces of constructible sheaves on surfaces. 2) Spaces of constructible sheaves on surfaces are spaces of Lagrangian branes (i.e. exact Lagrangians equipped with local systems) in symplectic 4-manifolds. 3) Spaces of Lagrangian branes in 4-manifolds are cluster varieties: fixing an exact Lagrangian surface and varying the local system carves out an algebraic torus, and surgeries on exact Lagrangians induce cluster transformations on these tori. Hence, lots of important geometric objects are cluster varieties. This is joint work with David Treumann, Vivek Shende, and Eric Zaslow

Nonorientable Surfaces and their Cluster Structure

Jon Wilson, Durham University

Dupont and Palesi defined quasi-triangulations of non-orientable surfaces. By looking at how lengths of arcs are related they gave birth to quasi-cluster algebras - an analogue of cluster algebras arising from non-orientable surfaces. I will link these algebras to Lam and Pylyavskyy's LP algebras, and remark on the structure of finite type quasi-cluster algebras.