Regular Languages

I used to hate writing assignments, but now I enjoy them.

I realized that the purpose of writing is to inflate weak ideas, obscure poor reasoning, and inhibit clarity.

With a little practice, writing can be an intimidating and impenetrable fog! Want to see my book report?

“THE DYNAMICS OF INTERBEING AND MONOLOGICAL IMPERATIVES IN DICK AND JANE: A STUDY IN PSYCHIC TRANSRELATIONAL GENDER MODES.”

ACADEMIA, HERE I COME!
Definition:

A language is a **Regular Language** iff some **Finite State Machine** recognizes it.
Intuition Building:

What would make a language NOT Regular?

Answer: It requires memory.
A Few Examples (not limited to):

Non-Regular languages

- $0^n1^n$
- $0^m1^n, m > n$
- $w = (0|1)^*$
- $ww$ or $ww^R$

where $\#_0(w) = \#_1(w)$

Regular Languages

- Contains a substring
- $0^*1^*0^+$
- Any finite language
- Binary numbers divisible by 3
Regular Operations

Let $A$ and $B$ be languages. We define the regular operations union, concatenation, and star as follows:

- **Union:** $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$.
- **Concatenation:** $A \circ B = \{ xy \mid x \in A \text{ and } y \in B \}$.
- **Star:** $A^* = \{ x_1x_2...x_k \mid k \geq 0 \text{ and each } x_i \in A \}$. 
Question: Are Regular Languages closed under Union, Concatenation, and Star?
Theorem: Regular Languages are Closed Under Union

Intuition:
- In terms of sets, what does Union mean?
- How can this be applied to two FSMs?
- Can you build a machine that models this behavior?

Define:
- $Q_3 = Q_1 \times Q_2$
  or, $Q_3 = \{(p, q) \mid p \in Q_1 \text{ and } q \in Q_2\}$
- $\Sigma$ (no change needed)
- $q_3 = (q_1, q_2)$
- $F_3 = (F_1 \times Q_2) \cup (Q_1 \times F_2)$
  Note: NOT $(F_1 \times F_2)$
- $\delta_3((p, q), a) = (\delta_1(p, a), \delta_2(q, a))$
  where $a \in \Sigma$ and $(p, q) \in Q_3$.

Construct $M_3 = M_1 \cup M_2$

What else would this proof need?