CSE 30151 & CSE 34151
Spring 2019
Homework 4
Due: February 28, 2019

Instructions: Homework must be typeset in LATEX. Proof solutions must be submitted according to the standard previously defined in class, unless otherwise specified.

1. (30 Points) You do not need to write an Intuition statement for any proof in this section. Hint: To help you understand part a, write down example strings.

   (a) Let $B = \{1^ky \mid y \in \{0,1\}^* \text{ and } y \text{ contains at least } k \text{ 1s, for } k \geq 1\}$. Show that $B$ is a regular language.

   (b) Let $C = \{1^ky \mid y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s, for } k \geq 1\}$. Show that $C$ isn’t a regular language.

2. (20 Points) Let $\Sigma = \{0,1,+,=\}$ and

   $$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$  

   Show that $ADD$ is not regular. You do not need to write an Intuition statement for this proof.

3. (35 Points) Let $x$ and $y$ be strings and let $L$ be any regular language. We say that $x$ and $y$ are distinguishable by $L$ if some string $z$ exists whereby exactly one of the strings $xz$ and $yz$ is a member of $L$; otherwise, for every string $z$, we have $xz \in L$ iff $yz \in L$ and we say that $x$ and $y$ are indistinguishable by $L$. If $x$ and $y$ are indistinguishable by $L$, we write $x \equiv_L y$. Write a proof showing that $\equiv_L$ is an equivalence relation (e.g., reflexive, symmetric, and transitive). $\equiv_L$ is $\equiv_{\text{equiv}_L}$.

4. (55 Points) Consider the language $F = \{a^ib^jc^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

   (a) Prove that $F$ is not regular. Hint: Your previous homework.

   (b) Show that $F$ acts like a regular language in the pumping lemma. In other words, give a pumping length $p$ and demonstrate that $F$ will always satisfy the three conditions of the pumping lemma for this value of $p$. Explain why this happens.

   (c) Explain why parts (a) and (b) do not contradict the pumping lemma.