General Instructions: Homework must be typeset in LATEX, and is due at the beginning of class on the due date. No late work is accepted.

1. (35 points) Let $\text{INFINITE}_{\text{PDA}} = \{\langle M \rangle | M \text{ is a PDA and } L(M) \text{ is an infinite language} \}$. Prove that $\text{INFINITE}_{\text{PDA}}$ is decidable. (Hint: See the solution provided for question 4.10 in the book. It will help you to understand how to solve this type of question.)

2. (35 points) Let $C = \{(G, x) | G \text{ is a CFG } x \text{ is a substring of some } y \in L(G) \}$. Prove that $C$ is decidable. (Hint: An elegant solution to this problem uses the decider for $E_{\text{CFG}}$.)

3. (35 points) A cosmetic state in a pushdown automaton is a state that is never used when processing any input string. Consider the problem of determining whether a pushdown automaton has any cosmetic states. Formulate this problem as the language $\text{COSMETIC}_{\text{PDA}}$ and prove that it is decidable.

4. (35 points) A cosmetic state in a Turing machine is a state that is never used when processing any input string. Consider the problem of determining whether a Turing machine has any cosmetic states. Formulate this problem as the language $\text{COSMETIC}_{\text{TM}}$ and show that it is undecidable.

5. (20 points) Find a match in the following instance of the Post Correspondence Problem:

$$\left\{ \left[ \begin{array}{c} 1 \\ 0 \end{array} \right], \left[ \begin{array}{c} 0 \end{array} \right], \left[ \begin{array}{c} 010 \\ 1 \end{array} \right], \left[ \begin{array}{c} 01 \\ 0101 \end{array} \right] \right\}$$
Helpful \LaTeX:

\begin{verbatim}
% Add this package to your document preamble.
% It allows you to use \text{} inside math mode.
% Try it without \text{}, and you will see the undesirable spacing problems.
% \usepackage{amsmath}
\textit{INFINITE}_PDA = \{ \langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language} \}\$

\[
\text{INFINITE}_PDA = \{ \langle M \rangle \mid M \text{ is a PDA and } L(M) \text{ is an infinite language} \}\$

% Create a custom macro called \post{}{}.\renewcommand{\post}[2]{\left\{ \frac{\tt#1}{\tt#2} \right\}}
\[ \left\{ \text{\post{1}{0}, \post{00}{0}, \post{010}{1}, \post{01}{0101} } \right\} \]

\[
\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 00 \\ 0 \end{bmatrix}, \begin{bmatrix} 010 \\ 1 \end{bmatrix}, \begin{bmatrix} 01 \\ 0101 \end{bmatrix} \right\}
\]

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