What Were We Talking About Last Time???

http://ozeda.com:8890

Infinity War spoiler but without context.
Proof of Equivalency

NFAs and DFAs

Proof by Construction:
A method to convert
\[ N = (Q, \Sigma, \delta, q_0, F) \]
into
\[ M = (Q', \Sigma, \delta', q'_0, F') \]
Convert NFA to DFA

Proof by Construction. (ignoring $\varepsilon$ for now)

Let $N = (Q, \Sigma, \delta, q_0, F)$ be a NFA that recognizes some language $A$.

Construct a DFA $M = (Q', \Sigma, \delta', q'_0, F')$ that recognizes the same language $A$.

$Q' = \mathcal{P}(Q)$

ex: $\{\{p_0\}, \{p_0, p_1\}, \{p_0, p_2\}, \{p_0, p_1, p_2\}, \ldots\}$

$q'_0 = \{q_0\}$

$F' = \{R \subseteq Q' | R$ contains an accept state of $N\}$

or, $F' = \{R \subseteq Q' | R \cap F \neq \emptyset\}$

$\delta'(R, a) = \{q \in Q | q \in \delta(r, a)$ for some $r \in R \}$

where $R \subseteq Q'$ and $a \in \Sigma$.

$\delta'(R, a) = \bigcup_{r \in R} \delta(r, a)$

For $\varepsilon$, define $E(R) = \{q | q$ can be reached from $R$ by traveling along 0 or more $\varepsilon$ arrows$\}$ where $R \subseteq Q$ and $q \in Q$.

$q'_0 = E(\{q_0\})$

$\delta'(R, a) = \{q \in Q | q \in E(\delta(r, a))$ for some $r \in R\}$
Practice: Convert this NFA to a DFA
Practice: Convert this NFA to a DFA
What we know

We defined DFAs.
We defined Computation using DFAs.
We defined Regular Languages using DFAs.
We defined NFAs.
We defined Computation using NFAs.
We showed equivalence of NFAs to DFAs.

Now What?
Regular Operations

They eat their fiber.

Union, Concatenation, and Star:
3 Proofs by Construction
Regular Operations: Union

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$, and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize $A_2$.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1 \cup A_2$.

$Q = \{q_0\} \cup Q_1 \cup Q_2$

$F = F_1 \cup F_2$

For $q \in Q$ and any $a \in \Sigma^*$,

$$
\delta(q, a) = \begin{cases} 
\delta_1(q, a) & q \in Q_1 \\
\delta_2(q, a) & q \in Q_2 \\
\{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\
\emptyset & q = q_0 \text{ and } a \neq \varepsilon
\end{cases}
$$
Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$, and $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ recognize $A_2$.

Construct $N = (Q, \Sigma, \delta, q_1, F_2)$ to recognize $A_1 \circ A_2$.

$$Q = Q_1 \cup Q_2$$

For $q \in Q$ and any $a \in \Sigma^*$,

$$\delta(q, a) = \begin{cases} 
\delta_1(q, a) & q \in Q_1 \text{ and } q \notin F_1 \\
\delta_1(q, a) & q \in F_1 \text{ and } a \neq \varepsilon \\
\delta_1(q, a) \cup \{q_2\} & q \in F_1 \text{ and } a = \varepsilon \\
\delta_2(q, a) & q \in Q_2
\end{cases}$$
Regular Operations: Star

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize $A_1$.

Construct $N = (Q, \Sigma, \delta, q_0, F)$ to recognize $A_1^*$.

$Q = \{q_0\} \cup Q_1$

$F = \{q_0\} \cup F_1$

For $q \in Q$ and any $a \in \Sigma$,

$$
\delta(q, a) = \begin{cases} 
\delta_1(q, a) & q \in Q_1 \text{ and } q \notin F_1 \\
\delta_1(q, a) & q \in F_1 \text{ and } a \neq \varepsilon \\
\delta_1(q, a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \\
\{q_1\} & q = q_0 \text{ and } a = \varepsilon \\
\emptyset & q = q_0 \text{ and } a \neq \varepsilon 
\end{cases}
$$