Review Session - Midterm #1

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1. The crime rate in a certain city can be approximated by a function $f(x, y, z)$, where x is the unemployment rate, y is the amount of social services available, and z is the size of the police force. Determine which of the following inequalities hold (explain your answer!):

   a) $\dfrac{\partial f}{\partial x} > 0$ or $\dfrac{\partial f}{\partial x} < 0$. b) $\dfrac{\partial f}{\partial y} > 0$ or $\dfrac{\partial f}{\partial y} < 0$. c) $\dfrac{\partial f}{\partial z} > 0$ or $\dfrac{\partial f}{\partial z} < 0$.

2. Find the values of x, y, z at which

$$f(x, y, z) = x^2 + 4y^2 + 5z^2 - 6x + 8y + 3$$

   assumes its minimum value.

3. Find he dimensions of a rectangular box of volume 1000 cubic inches for which the sum of the dimensions is minimized.

4. Calculate $\displaystyle\int_1^4 \left(\int_x^{x^2} xy dy \right) dx$, and $\displaystyle\int_0^1 \left(\int_0^4 (x\sqrt{y} + y) dy \right) dx$.

5. Let R be the rectangle consisting of all points (x, y) such that $0 \leq x \leq 2$, $2 \leq y \leq 3$. Calculate $\displaystyle\iint_R e^{-x-y}\, dx\, dy$.

6. Find t such that $-\pi/2 \leq t \leq \pi$ and t satisfies

   a) $\sin(t) = -\sin(3\pi/8)$. b) $\sin(t) = -\cos(t)$. c) $\sin(t) = -\cos(\pi/3)$.

7. Determine the value of $\sin(t)$ when $t = 5\pi$, -2π, $17\pi/2$, $-13\pi/2$.

8. A tree casts a 60-foot shadow when the angle of elevation of the sun (measured from the horizontal) is 53°. How tall is the tree?

9. Differentiate $y = \tan(x^4 + x^2)$, $y = \ln(x)\cos(x)$, $f(t) = \dfrac{\tan(2t)}{\cos(t)}$, $f(t) = e^{\tan(t)}$, $y = e^{3x}\sin^4(x)$.

10. Use the identity $1 + \tan^2(t) = \sec^2(t)$ to evaluate $\displaystyle\int_0^{\pi/4} \tan^2(x) dx$.