Worksheet 11

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1. Use a second Taylor polynomial at $t = 0$ to estimate the area under the graph of

$$y = -\ln(\cos(2t))$$

between $t = 0$ and $t = 1/2$.

2. Let $p_4(x)$ be the fourth Taylor polynomial of $f(x) = e^x$ at $x = 0$. Show that the error in using $p_4(.1)$ as an approximation for $e^{.1}$ is at most $2.5 \cdot 10^{-7}$.

3. Find $\displaystyle\sum_{k=0}^{\infty} \frac{3^k + 5^k}{7^k}$.

4. Determine if the series

$$\sum_{k=0}^{\infty} \frac{k^3}{(k^4+1)^2}$$

is convergent or divergent.

5. Find the Taylor series expansion at $x = 0$ of

$$\int e^{x^3} dx \quad \text{and} \quad \int x e^{x^2} dx.$$

6. For any positive number k, verify that $f(x) = \dfrac{1}{2k^3} \cdot x^2 e^{-x/k}$, $x \geq 0$, is a density function. Let X be a random variable with density function $f(x)$. Determine the probability that X takes a value larger than 5.

7. The lifetime of a certain TV picture tube is an exponential random variable with an expected value of 5 years. The tube manufacturer sells the tube for \$100, but will give a complete refund if the tube burns out within 3 years. Then the revenue that the manufacturer receives on each tube is a discrete random variable Y, with values 100 and 0. Determine the expected revenue per tube.