Worksheet 8

Claudiu Raicu

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(1-6) Test the series for convergence

1. $\displaystyle\sum_{k=1}^{\infty} \frac{2k+1}{k^2+k+2}$.

2. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{2^k+k}$.

3. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2\ln(k)}$.

4. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln^2(k)}$.

5. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ln(k)}$.

6. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5^k}\cdot\cos^2(k\pi/4)$.

7. Determine the Taylor expansion at 0 of $\dfrac{x}{(1-x)^3}$.

8. Determine the Taylor series at $x=0$ of the function $x\cdot\sin(x^2)$. You may use the fact that the Taylor series of $\cos(x)$ is

$$\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

9. Let $f(x) = e^{x^2}$. Determine $f^{(6)}(0)$ and $f^{(13)}(0)$.