Math 16B, Spring '11
Quiz 11, April 12

1. Determine $f^{(4)}(0)$ and $f^{(5)}(0)$, where
$$f(x) = \frac{1}{(1-x^2)^2}.$$

Solution. We first note that $f(x) = g(x^2)$, where
$$g(x) = \frac{1}{(1-x)^2} = \left(\frac{1}{1-x}\right)' = (1 + x + x^2 + x^3 + \cdots)' = 1 + 2x + 3x^2 + 4x^3 + \cdots .$$
It follows that
$$f(x) = g(x^2) = 1 + 2x^2 + 3x^4 + 4x^6 + \cdots$$
Since the coefficient of x^4 in the Taylor series of f is 3, it follows that
$$\frac{f^{(4)}(0)}{4!} = 3, \text{ i.e. } \mathrm{f}^{(4)}(0) = 3 \cdot 4! = 3 \cdot 24 = 72.$$
Since the coefficient of x^5 in the Taylor series of $f(x)$ is zero, it follows that
$$\frac{f^{(5)}(0)}{5!} = 0, \text{ i.e. } \mathrm{f}^{(5)}(0) = 0.$$
□

2. (5 points) The number of accidents per week at a busy intersection was recorded for a year. There were 13 weeks with no accidents, 26 weeks with one accident, 13 weeks with two accidents. A week is to be selected at random and the number of accidents noted. Let X be the outcome. Then X is a random variable taking on the values $0, 1, 2$.

(a) Write out a probability table for X.

(b) Compute the expected value, variance, and standard deviation of X.

Solution. (a) Since $X(0) = 13/52 = 1/4$, $X(1) = 26/52 = 1/2$ and $X(2) = 13/52 = 1/4$, the probability table for X is

X = accidents/week	0	1	2
probability	1/4	1/2	1/4

(b) The average number of accidents in a week, i.e. the expected value $E(X)$ is given by
$$E(X) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 1.$$
We now draw the probability table for $X - E(X)$:

X − E(X)	−1	0	1
probability	1/4	1/2	1/4

It follows that the variance of X is given by
$$\mathrm{Var(X)} = (-1)^2 \cdot \frac{1}{4} + 0^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{4} = \frac{1}{2}.$$
The standard deviation is
$$\sqrt{\mathrm{Var(X)}} = \frac{1}{\sqrt{2}}.$$
□