Math 16B, Spring '11
Quiz 12, April 12

1. (5 points) Verify that $f(x) = \frac{8}{9} \cdot x$, $0 \leq x \leq \frac{3}{2}$ is a probability density function. Find $\Pr(1 \leq X)$ when X is a random variable whose density function is $f(x)$.

   Solution. We have
$$\int_0^{3/2} f(x)dx = \int_0^{3/2} \frac{8x}{9} dx = \left[\frac{8x^2}{2 \cdot 9}\right]_0^{3/2} = 1,$$
   hence $f(x)$ is a probability density function. We have
$$\Pr(1 \leq X) = \int_1^{3/2} f(x)dx = \left[\frac{4x^2}{9}\right]_1^{3/2} = 1 - \frac{4}{9} = \frac{5}{9}.$$
   □

2. (5 points) The amount in time (in minutes) that a person spends reading the editorial page of the newspaper is a random variable with the density function $f(x) = \frac{1}{72} \cdot x$, $0 \leq x \leq 12$. Find the average time spent reading the editorial page.

   Solution. The average time spent reading the editorial page is the expected value of the random variable X whose density function is $f(x)$, i.e.
$$\int_0^{12} x \cdot \frac{x}{72} dx = \int_0^{12} \frac{x^2}{72} dx = \left[\frac{x^3}{3 \cdot 72}\right]_0^{12} = \frac{12^3}{3 \cdot 6 \cdot 12} = \frac{12}{3} \cdot \frac{12}{6} \cdot \frac{12}{12} = 4 \cdot 2 \cdot 1 = 8 \text{ minutes.}$$
   □