Math 16B, Spring '11
Quiz 1, January 25

1. (5 points) Draw the level curve of the function $f(x, y) = x - y$ containing the point $(0, 0)$.

   Solution. The level curve $f(x, y) = K$ passes through $(0, 0)$ if and only if $f(0, 0) = K$. Since $f(0, 0) = 0$, we must have $K = 0$. The level curve is therefore the line $x = y$ (draw it!).

   □

2. (5 points) Let $f(x, y) = e^{xy} + x^4y + y^3$. Find $\dfrac{\partial^2 f}{\partial x^2}$ and $\dfrac{\partial^2 f}{\partial y \partial x}$.

   Solution. We first find $\dfrac{\partial f}{\partial x}$:

   $$\frac{\partial f}{\partial x} = ye^{xy} + 4x^3y. \qquad (*)$$

   Taking the derivative with respect to x of the function above, we get

   $$\frac{\partial^2 f}{\partial x^2} = y^2e^{xy} + 12x^2y.$$

   Taking the derivative with respect to y in (*) (use the product rule to compute the derivative of ye^{xy} with respect to y !!), we get

   $$\frac{\partial^2 f}{\partial y \partial x} = e^{xy} + xye^{xy} + 4x^3.$$

   □