Math 16B, Spring '11
Quiz 3, February 8

1. (5 points) Calculate the volume over the region R bounded above by the graph of $f(x,y) = x^2 + y^2$, where R is the rectangle bounded by the lines $x = 1, x = 3$, $y = 0$ and $y = 1$.

Solution. The volume we seek can be computed as the iterated integral

$$\int_1^3 \left(\int_0^1 (x^2 + y^2) dy \right) dx.$$

We have

$$\int_0^1 (x^2 + y^2) dy = \left[x^2 y + \frac{y^3}{3} \right]_0^1 = x^2 + \frac{1}{3},$$

and

$$\int_1^3 (x^2 + \frac{1}{3}) dx = \left[\frac{x^3}{3} + \frac{x}{3} \right]_1^3 = \frac{3^3 + 3}{3} - \frac{1^3 + 1}{3} = \frac{28}{3}.$$

□

2. (5 points) Find t such that $0 \leq t \leq \pi$ and t satisfies

a) $\cos(t) = \cos(5\pi/4)$.

b) $\cos(t) = \sin(-5\pi/8)$.

Solution. a) We have that $\pi < 5\pi/4 < 2\pi$. Using the formula $\cos(x) = \cos(2\pi - x)$ allows us to move any x lying between π and 2π into the interval $(0, \pi)$:

$$\cos(5\pi/4) = \cos(2\pi - 5\pi/4) = \cos(3\pi/4),$$

hence $t = 3\pi/4$.

b) We first go from the sine value to the cosine value, using the formula $\sin(x) = \cos(\pi/2 - x)$:

$$\sin(-5\pi/8) = \cos(\pi/2 + 5\pi/8) = \cos(9\pi/8).$$

Now $\pi < 9\pi/8 < 2\pi$, so we can use the same trick as in part a):

$$\cos(9\pi/8) = \cos(2\pi - 9\pi/8) = \cos(7\pi/8),$$

hence $t = 7\pi/8$.

□