Math 16B, Spring ’11
Quiz 4, February 22

1. (5 points) Make an appropriate substitution and evaluate the integral
$$\int \cos^3(x)\sin(x)dx.$$
Solution. The substitution $u = \cos(x)$ yields $du = -\sin(x)dx$ and
$$\int \cos^3(x)\sin(x)dx = \int u^3 \cdot (-1)du = \frac{-u^4}{4} + C = -\frac{\cos^4(x)}{4} + C.$$
$\square$

2. (5 points) Evaluate the integral
$$\int \sqrt{x}\ln(\sqrt{x})dx.$$
Solution. We first note that $\ln(\sqrt{x}) = \ln(x)/2$. We use integration by parts with
$$f(x) = \ln(\sqrt{x}) = \ln(x)/2, \quad g(x) = \sqrt{x}.$$
This yields
$$f'(x) = \frac{1}{2x}, \quad G(x) = \frac{2x^{3/2}}{3},$$
whence
$$\int \sqrt{x}\ln(\sqrt{x})dx = \ln(\sqrt{x}) \cdot \frac{2x^{3/2}}{3} - \int \frac{1}{2x} \cdot \frac{2x^{3/2}}{3}dx = \frac{x^{3/2}\ln(x)}{3} - \int \frac{\sqrt{x}}{3}dx$$
$$= \frac{x^{3/2}\ln(x)}{3} - \frac{2x^{3/2}}{9} + C = \frac{(3\ln(x) - 2)x^{3/2}}{9} + C.$$
$\square$