Math 16B, Spring '11
Quiz 6, March 8

1. (5 points) Solve the differential equation

$$y' = t^2 e^{-3y},$$

with the initial condition

$$y(0) = 2.$$

Solution. We rewrite the equation as

$$\frac{dy}{dt} = t^2 e^{-3y},$$

separate the variables

$$e^{3y} dy = t^2 dt,$$

and integrate

$$\int e^{3y} dy = \int t^2 dt,$$

i.e.

$$\frac{e^{3y}}{3} = \frac{t^3}{3} + C.$$

The initial condition $y(0) = 2$ implies

$$C = \frac{e^6}{3},$$

so

$$\frac{e^{3y}}{3} = \frac{t^3 + e^6}{3}$$

which we can rewrite as

$$e^{3y} = t^3 + e^6 \Leftrightarrow y = \frac{1}{3} \ln(t^3 + e^6).$$

□

2. (5 points) Solve the initial-value problem

$$ty' + y = \sin(t), \ y\left(\frac{\pi}{2}\right) = 0, \ t > 0.$$

Proof. We observe that the left hand side is precisely the derivative of $t \cdot y$, so we can rewrite the equation as

$$(t \cdot y)' = \sin(t).$$

Integrating, we obtain

$$t \cdot y = -\cos(t) + C.$$

The initial condition $y(\pi/2) = 0$ yields $C = 0$, thus

$$y = \frac{-\cos(t)}{t}.$$

□