Math 16B, Spring '11
Quiz 9, April 5

1. (5 points) If

$$f(x) = 2 - 6(x-1) + \frac{3}{2!}(x-1)^2 - \frac{5}{3!}(x-1)^3 + \frac{1}{4!}(x-1)^4,$$

what are $f''(1)$ and $f'''(1)$?

Solution. Since $f(x)$ is a polynomial of degree 4, it is equal to its fourth Taylor polynomial. We get that

$$f(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4,$$

yielding $f''(1) = 3$ and $f'''(1) = -5$. □

2. (3 points) (a) Give an example of a geometric series that is convergent. Calculate its sum.

(2 points) (b) Give an example of a geometric series that is not convergent (explain).

Solution. (a) We take $a = 1$ and $r = 1/2$. The geometric series

$$a + ar + ar^2 + \cdots = \sum_{k=0}^{\infty} \frac{1}{2^k}$$

is convergent because $|r| < 1$, and its sum is

$$\frac{a}{1-r} = \frac{1}{1-1/2} = 2.$$

(b) We take $a = 1$ and $r = -1$. The geometric series

$$a + ar + ar^2 + \cdots = \sum_{k=0}^{\infty} (-1)^k = 1 + (-1) + 1 + (-1) + 1 + (-1) + \cdots$$

is divergent because $|r| \geq 1$. Alternatively, we see that the partial sums of the series are

$$\begin{aligned} 1 &= 1, \\ 1 + (-1) &= 0, \\ 1 + (-1) + 1 &= 1, \\ 1 + (-1) + 1 + (-1) &= 0, \end{aligned}$$

i.e. they alternate between 1 and 0, which means they form a divergent sequence.

□