

Worksheet 17

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1. Verify that $y = \sin x \cos x - \cos x$ is a solution of the initial-value problem

$$y' + (\tan x)y = \cos^2 x, \quad y(0) = -1$$

on the interval $-\pi/2 < x < \pi/2$.

2. (a) For what values of k does the function $y = \cos kt$ satisfy the differential equation $4y'' = -25y$?

(b) For those values of k , verify that every member of the family of functions $y = A \sin kt + B \cos kt$ is also a solution.

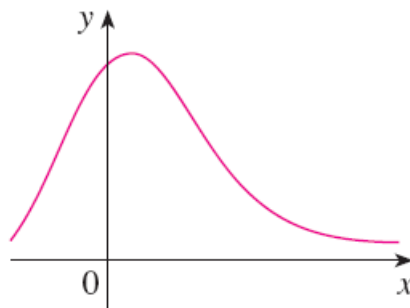
3. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^4 - 6y^3 + 5y^2.$$

(a) What are the constant solutions of the equation?

(b) For what values of y is y increasing/decreasing?

4. The function with the given graph is a solution of one of the following differential equations. Decide which is the correct equation and justify your answer.

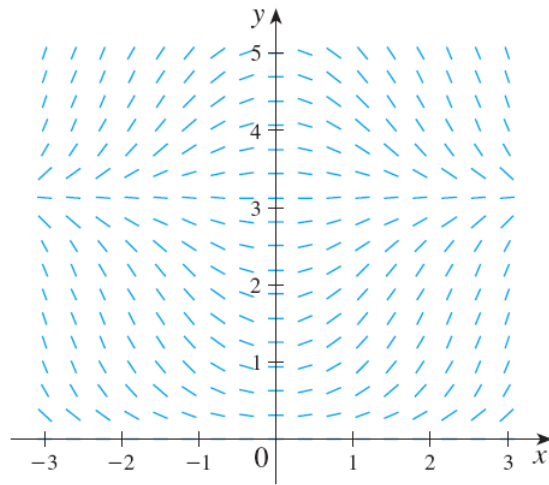


- (a) $y' = 1 + xy$. (b) $y' = 2xy$. (c) $y' = 1 - 2xy$.

5. A direction field for the differential equation $y' = x \sin y$ is shown.

(i) Sketch the graphs of the solutions that satisfy the given initial conditions.

- (a) $y(0) = 1$. (b) $y(0) = 2$. (c) $y(0) = \pi$. (d) $y(0) = 4$. (e) $y(0) = 5$.



(ii) Find all the equilibrium solutions.

6. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = 1 - xy$, $y(0) = 0$.